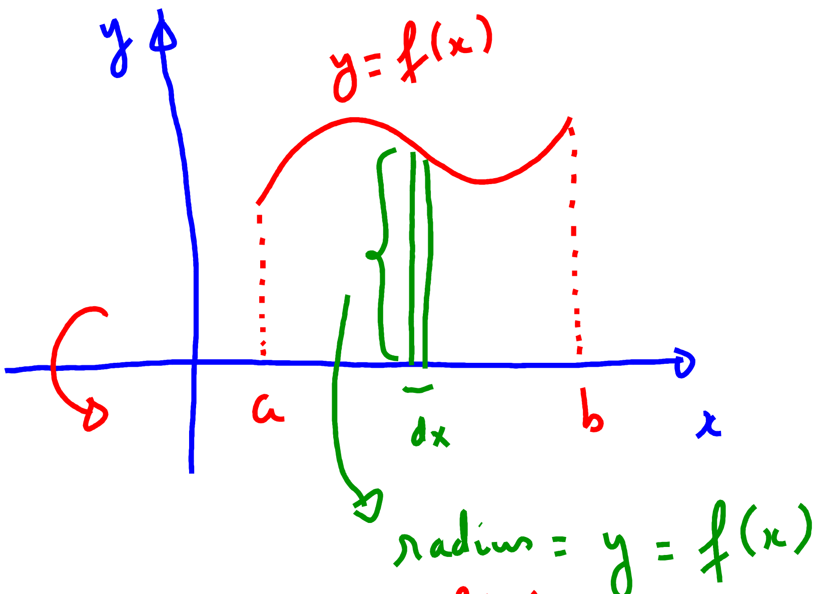
2.3 Volumes by Cylindrical Shell.

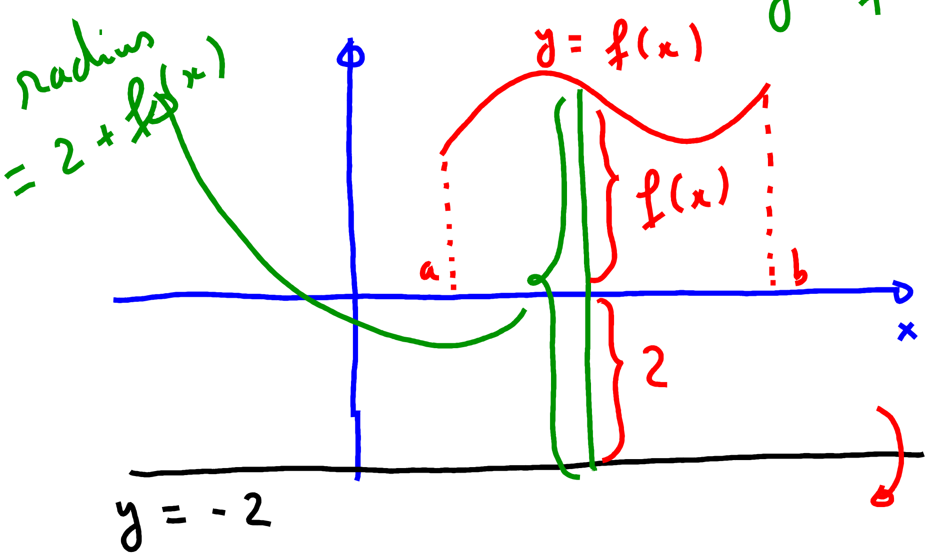
Recall: Disk method



Volume of Solid obtained

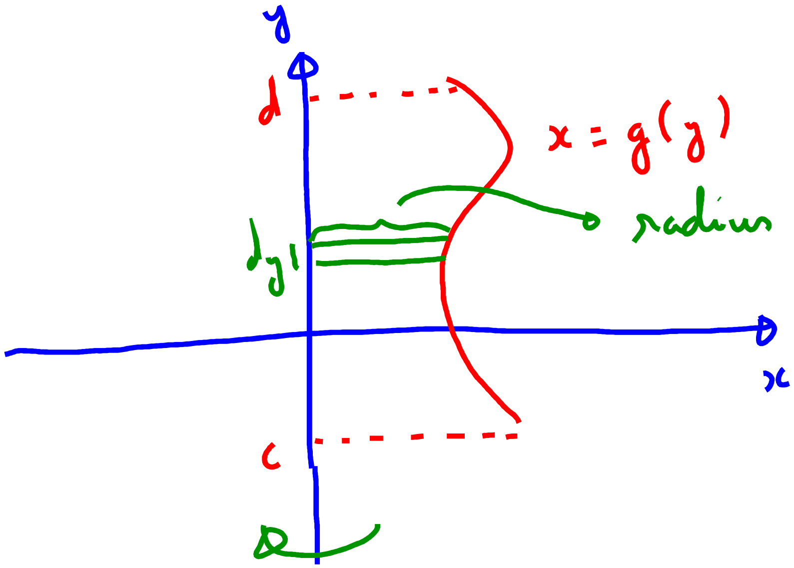
$$= \int_a^b \pi \cdot (\text{radius})^2 \, dx$$

$$\boxed{V = \pi \cdot \int_a^b [f(x)]^2 \, dx}$$



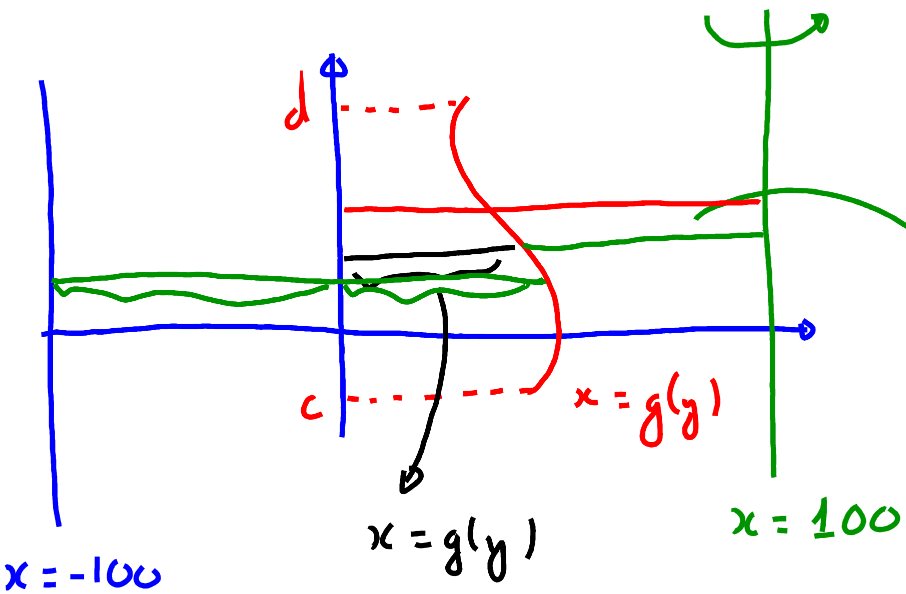
$$V = \int_a^b \pi \cdot (\text{radius})^2 \, dx$$

$$\boxed{V = \pi \int_a^b [2 + f(x)]^2 \, dx}$$



radius = x = g(y)

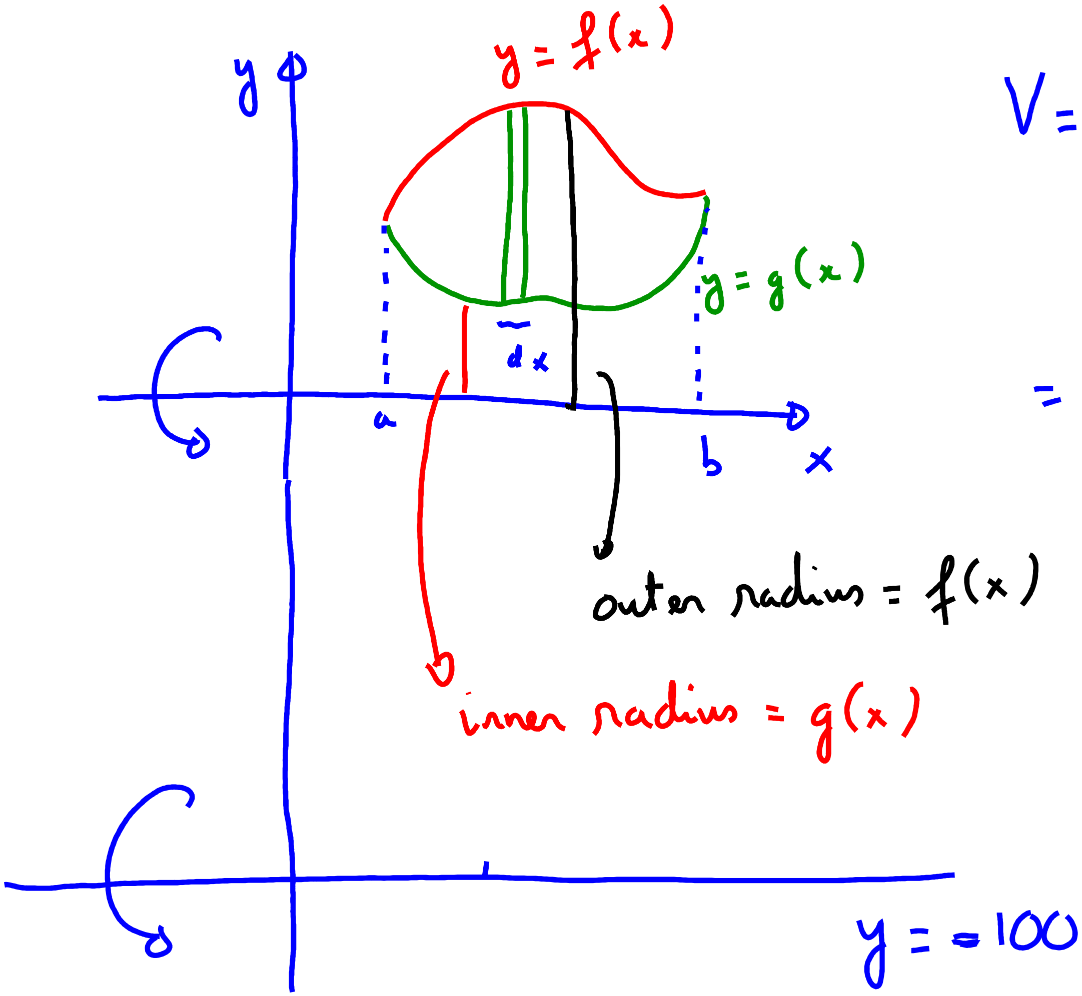
$$V = \pi \cdot \int_c^d [g(y)]^2 \, dy$$



$$V = \pi \cdot \int_c^d [100 - g(y)]^2 \, dy$$

radius = 100 - g(y)

Washer Method



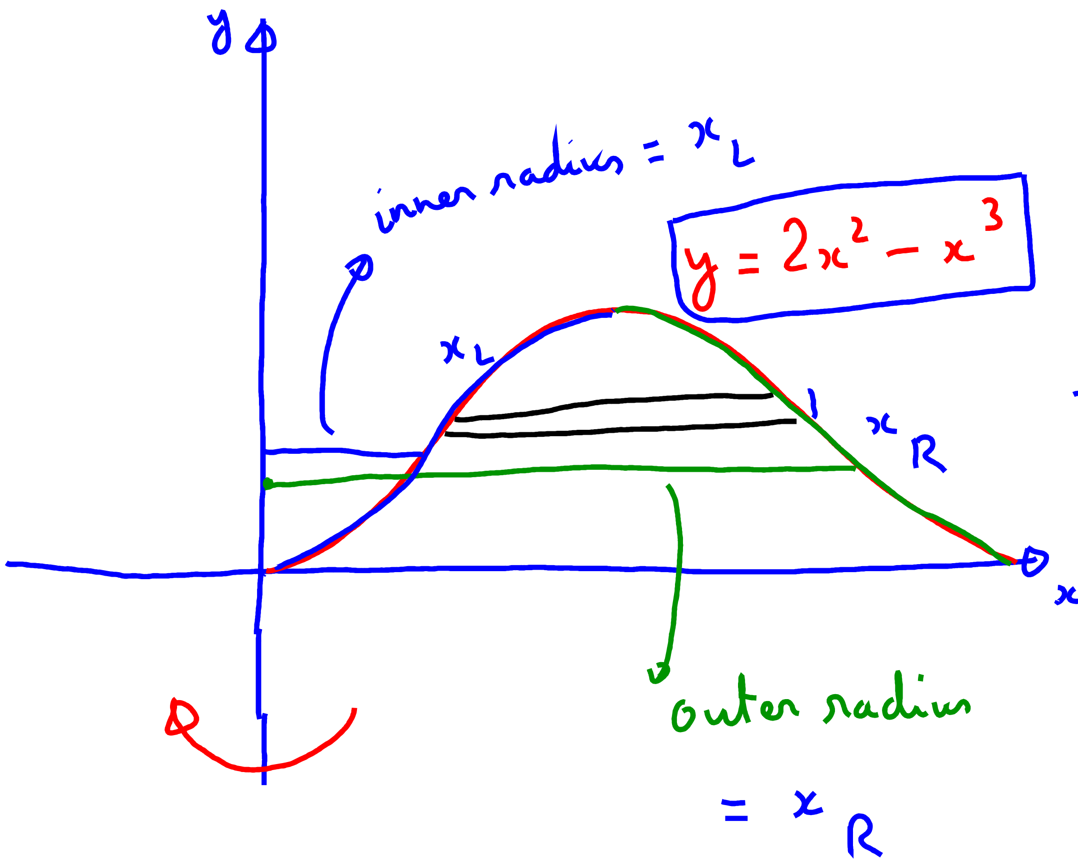
$$V = \int_a^b \pi \left[(\text{outer radius})^2 - (\text{inner radius})^2 \right] dx$$

$$= \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

$$V = \pi \int_a^b \left([100 + f(x)]^2 - [100 + g(x)]^2 \right) dx$$

<u>Key</u>: For Disk/Washer method, the slices are perpendicular to the axis of rotation.

E.g. for which the disk/washer method does not work very well.



Volume of solid obtained by rotating this region about the y-axis.

Try Washer:

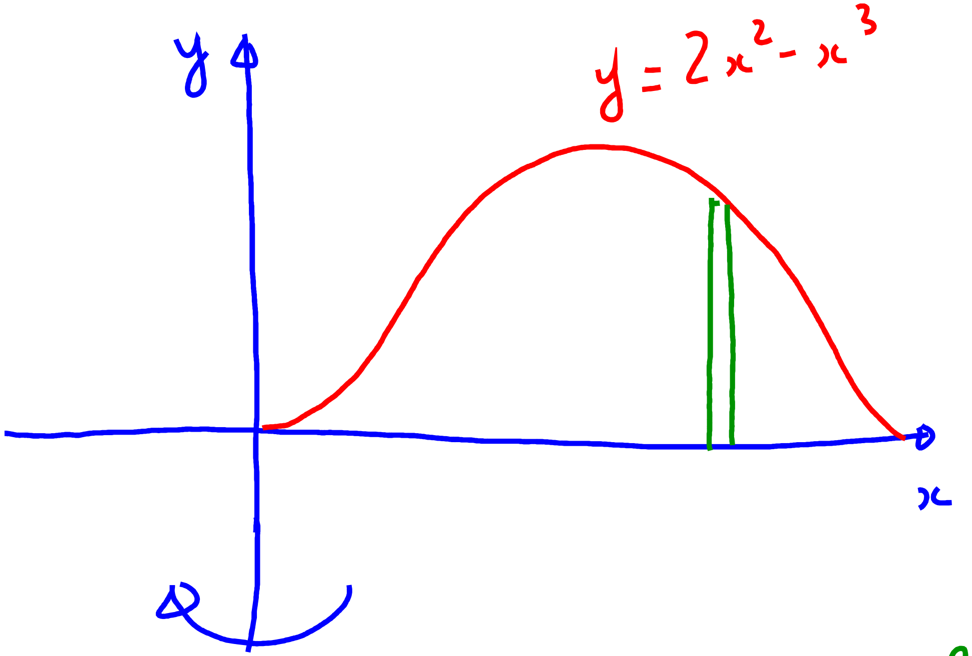
$$V = \pi \int \left[(\text{outer radius})^2 - (\text{inner radius})^2\right] dy$$

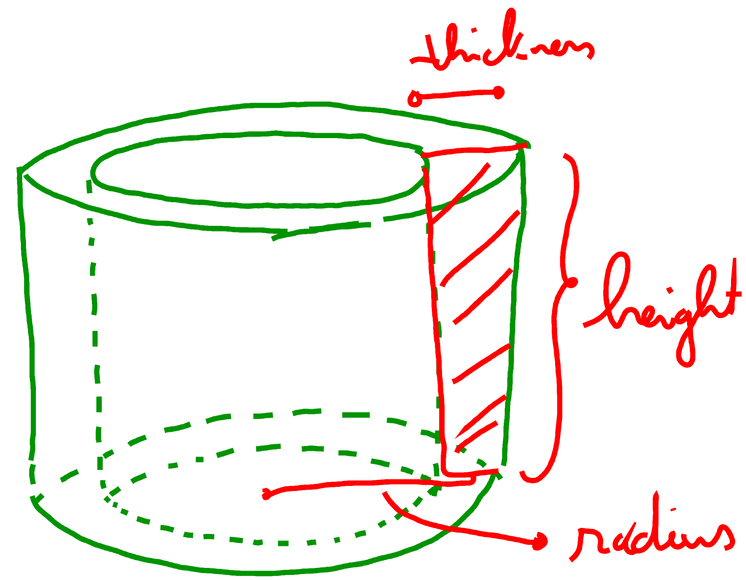
Solve for x in terms of y from the given equation

$\longrightarrow$ Shell Method will be easier.

Difference between disk / washer and shell method:

In shell method slices are parallel to the axis of rotation.





What happens if we rotate that slice about the y axis?

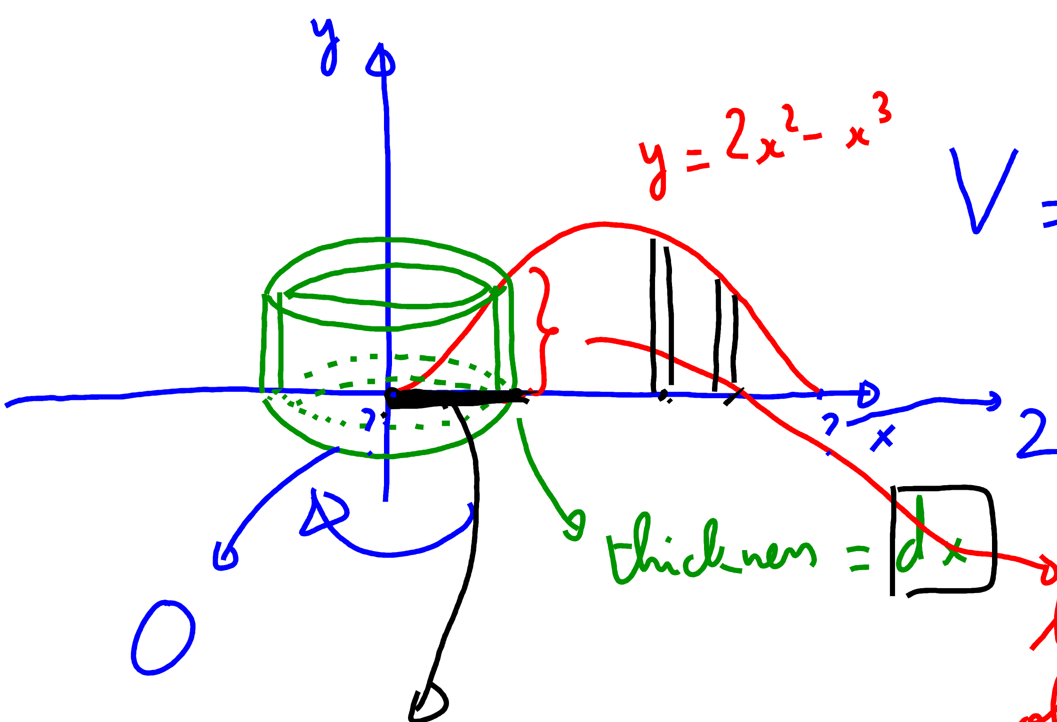
shell

Volume of solid

= Sum of volume of these small shells

(Integral)

Volume of a shell = $\underline{2\pi \cdot (\text{radius of shell}) \cdot (\text{height}) \cdot (\text{thickness})}$



$$V = \int_0^2 \underbrace{2\pi \cdot x \cdot (2x^2 - x^3) \cdot dx}_{\text{Volume of a single shell}}$$

thickness = $\boxed{dx}$

height of shell = y = $\boxed{2x^2 - x^3}$

Radius = $\boxed{x}$

Bounds for integral: $2x^2 - x^3 = 0$

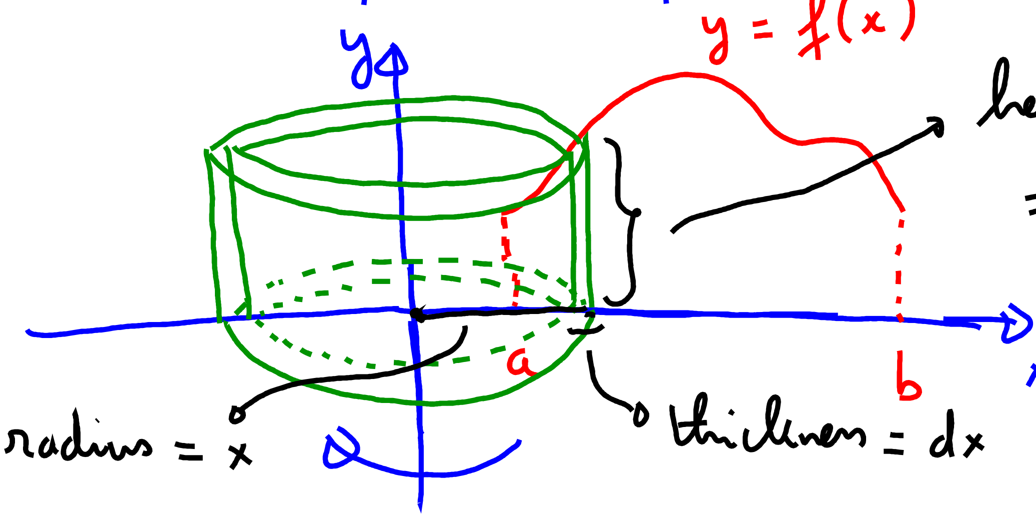
$x^2(2 - x) = 0$; $x = 0$; $x = 2$

$$V = 2\pi \int_0^2 x \cdot (2x^2 - x^3)\,dx$$

$$= 2\pi \int_0^2 (2x^3 - x^4)\,dx = 2\pi \cdot \left(2 \cdot \frac{x^4}{4} - \frac{x^5}{5}\right)\Bigg|_0^2$$

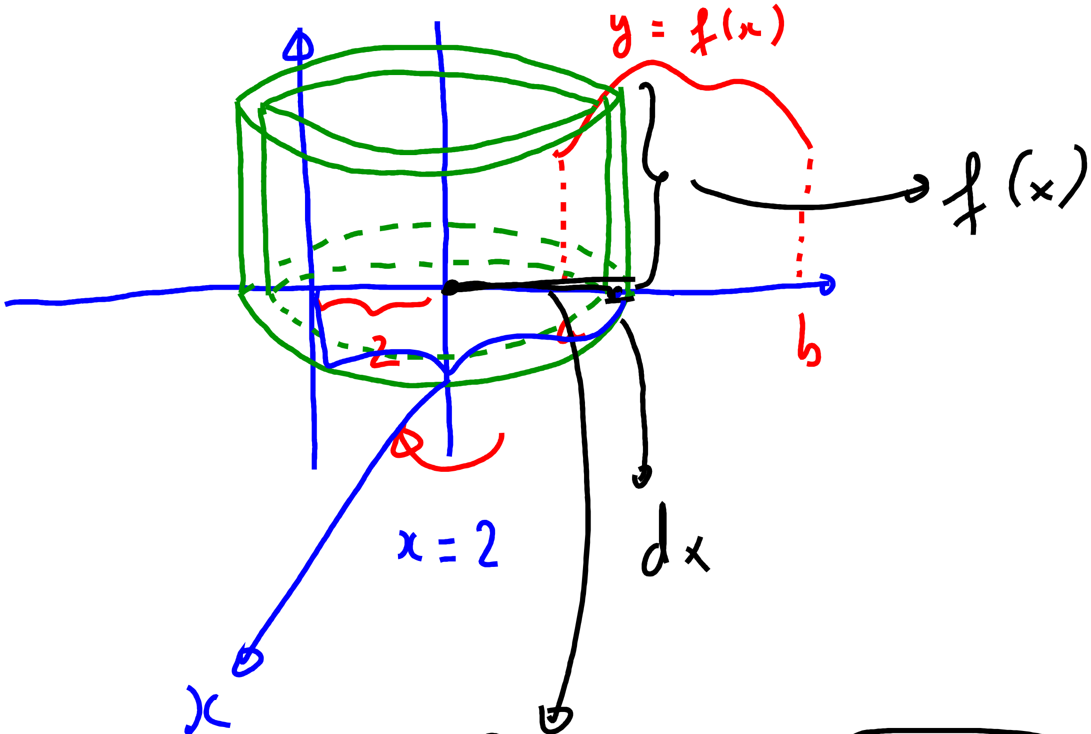
$$= 2\pi \cdot \left(\frac{x^4}{2} - \frac{x^5}{5}\right)\Bigg|_0^2 = 2\pi \cdot \left(8 - \frac{32}{5}\right) = \boxed{\frac{16\pi}{5}}$$

General formula for the shell method



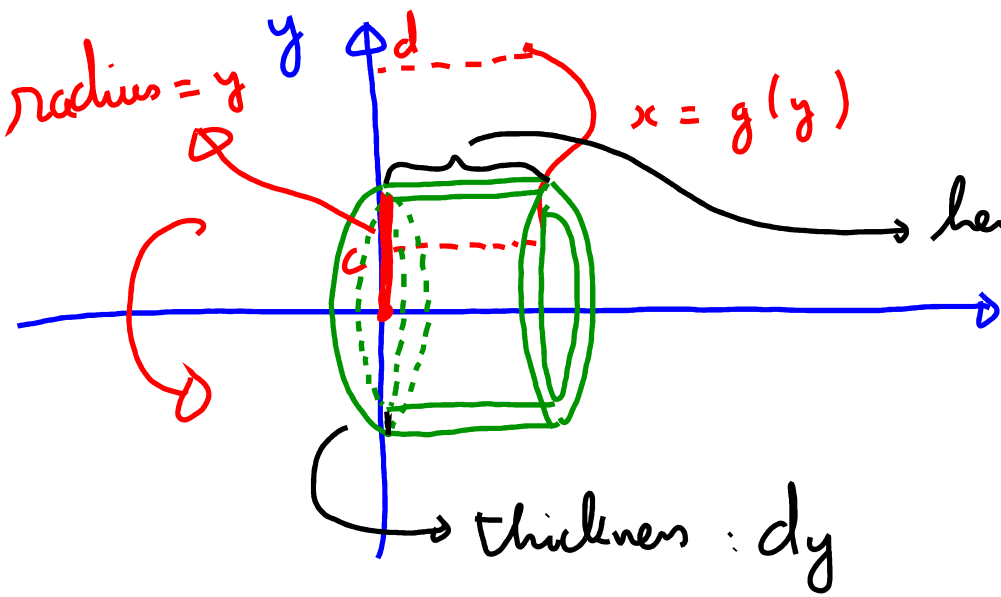
height of shell $= y = f(x)$

$$V = \int_a^b \underbrace{2\pi x \cdot f(x) \cdot dx}_{\text{Volume of 1 shell}}$$



$$V = 2\pi \int_a^b (x-2)\cdot f(x)\,dx$$

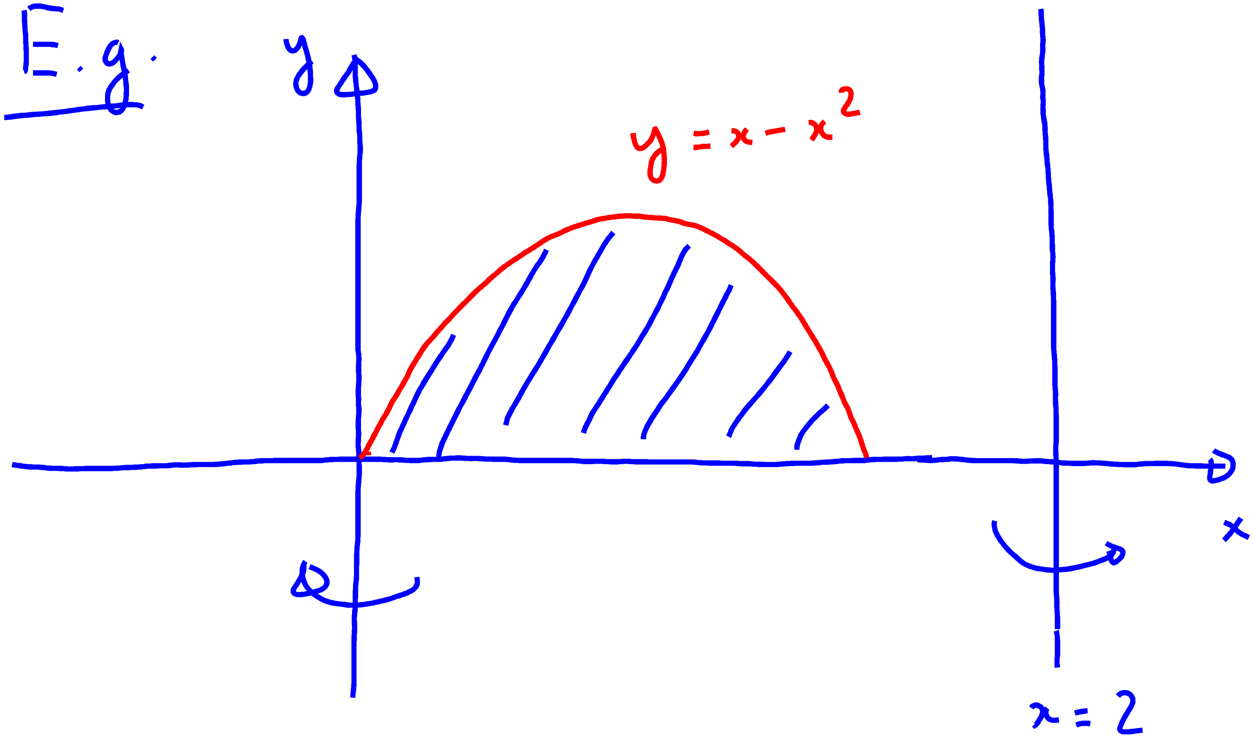
Radius of shell $= ?$ $\boxed{x-2}$



height $= x = g(y)$

$$V = 2\pi \int_c^d y\cdot g(y)\cdot dy$$

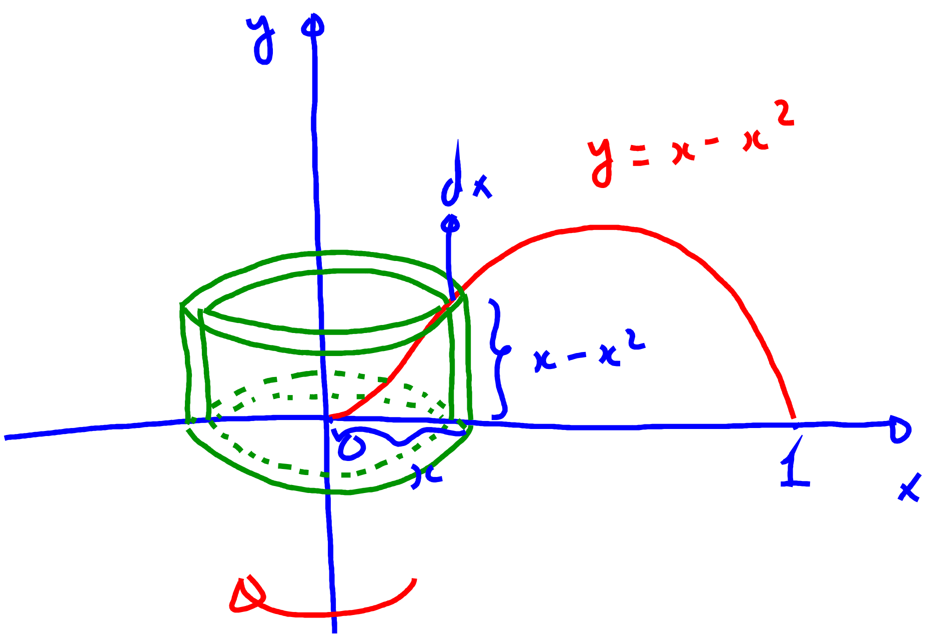
E.g.



① Find the volume of the solid obtained when the shaded region is rotated about y-axis

② Find the volume of solid obtained when it is rotated about the line $x = 2$.

1



$$V = 2\pi \cdot \int_0^1 x \cdot (x - x^2)\, dx$$

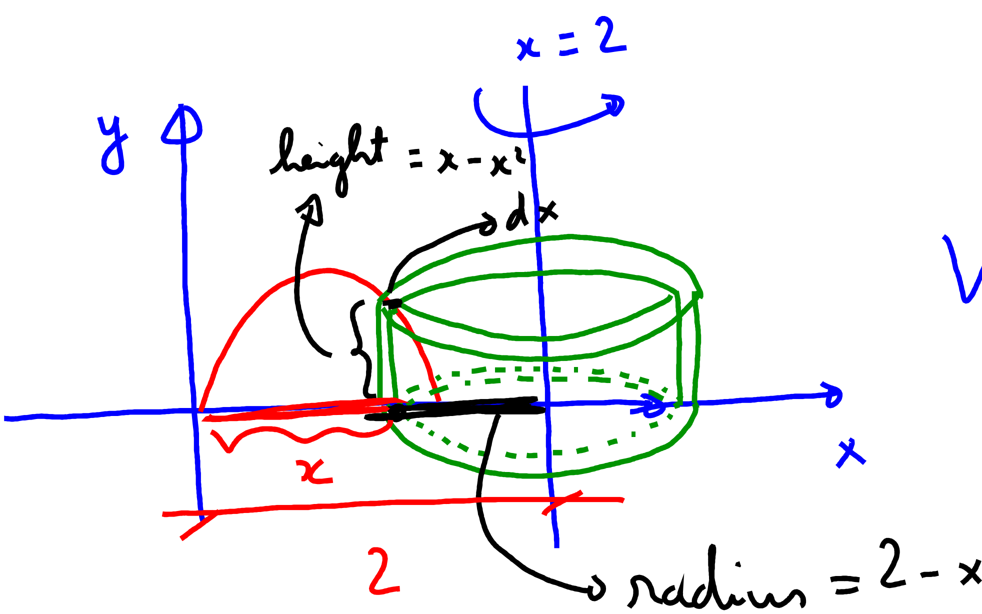
$$= 2\pi \cdot \int_0^1 (x^2 - x^3)\, dx$$

$$= 2\pi \cdot \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{4}\right) = \boxed{\frac{\pi}{6}}.$$

x- intercepts : $x - x^2 = 0$

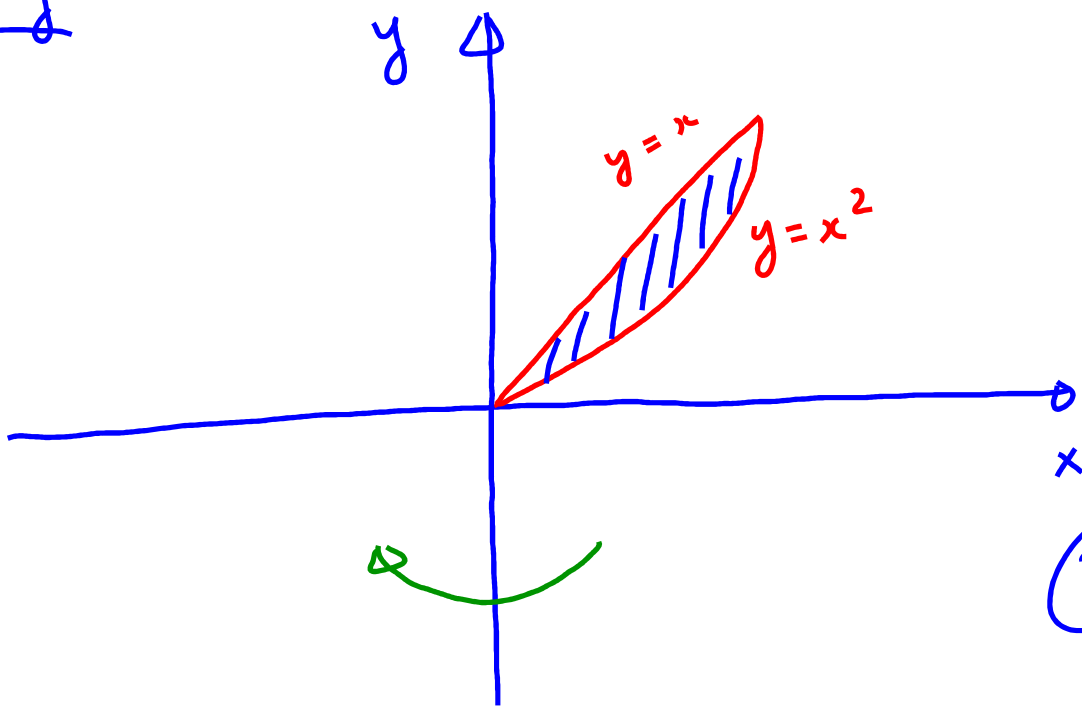
$x(1-x) = 0$; $x = 0$; $x = 1$



$$V = 2\pi \cdot \int_0^1 (2 - x)(x - x^2)\, dx$$

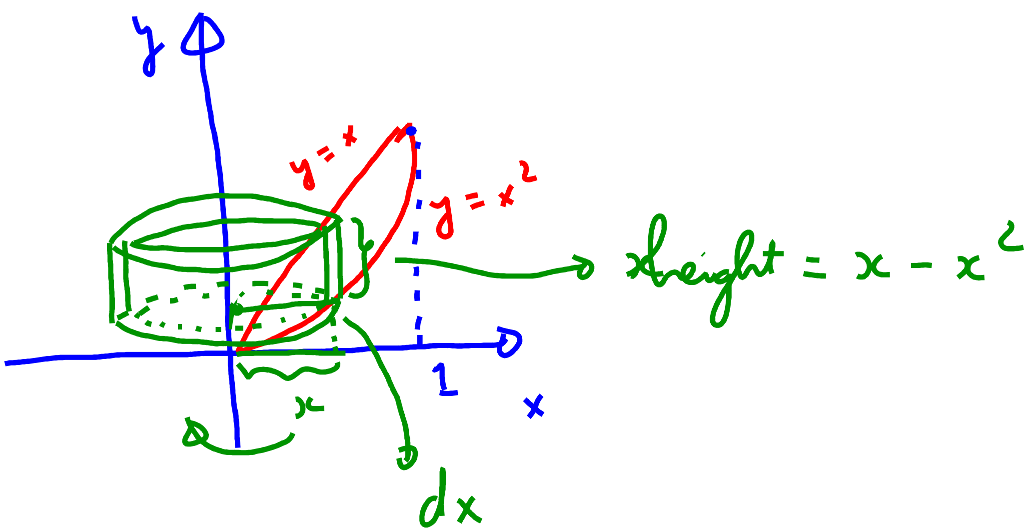
$$= 2\pi \int_0^1 (2x - 3x^2 + x^3)\, dx = \boxed{\frac{\pi}{2}}.$$

E.g.



① Find volume of solid obtained by rotating shaded region about y-axis by shell method

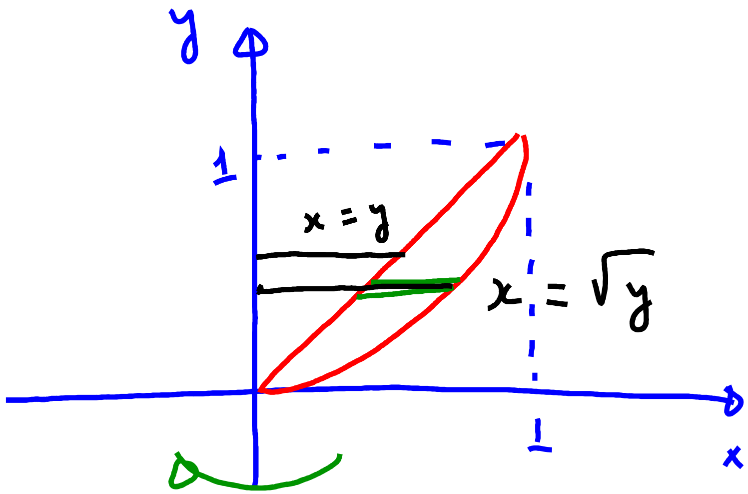
② Find the volume by washer method.



$$V = 2\pi \int_0^1 x \cdot (x - x^2) \cdot dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)\, dx$$

$$= \boxed{\frac{\pi}{6}}$$



$$V = \pi \int_0^1 (\text{outer radius})^2 - (\text{inner radius})^2 \cdot dy$$

$$= \pi \int_0^1 (\sqrt{y})^2 - y^2 \, dy$$

$$= \pi \int_0^1 (y - y^2) \, dy = \boxed{\frac{\pi}{6}} .$$