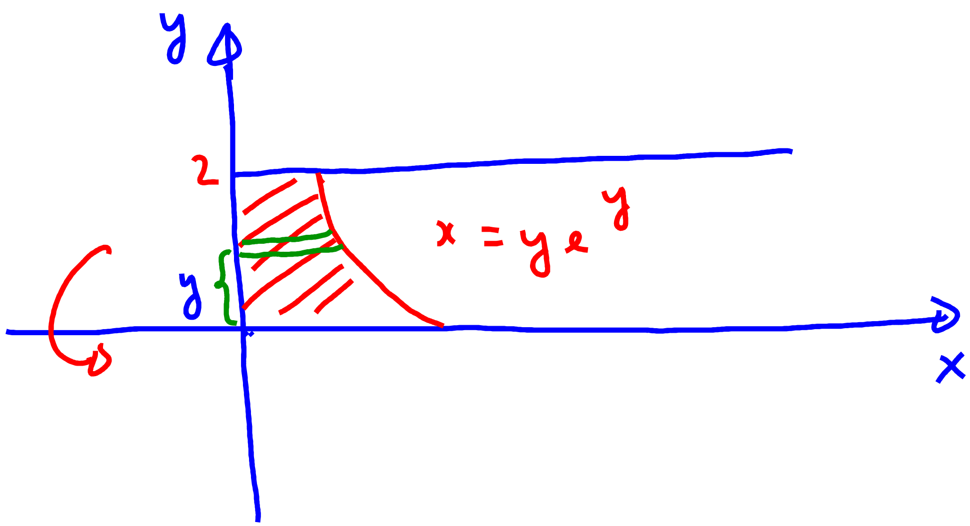
2.5. #12

Use shell to find volume:

Region: $x = y e^{y}$; $y=0$; $y=2$.

Rotate about x-axis

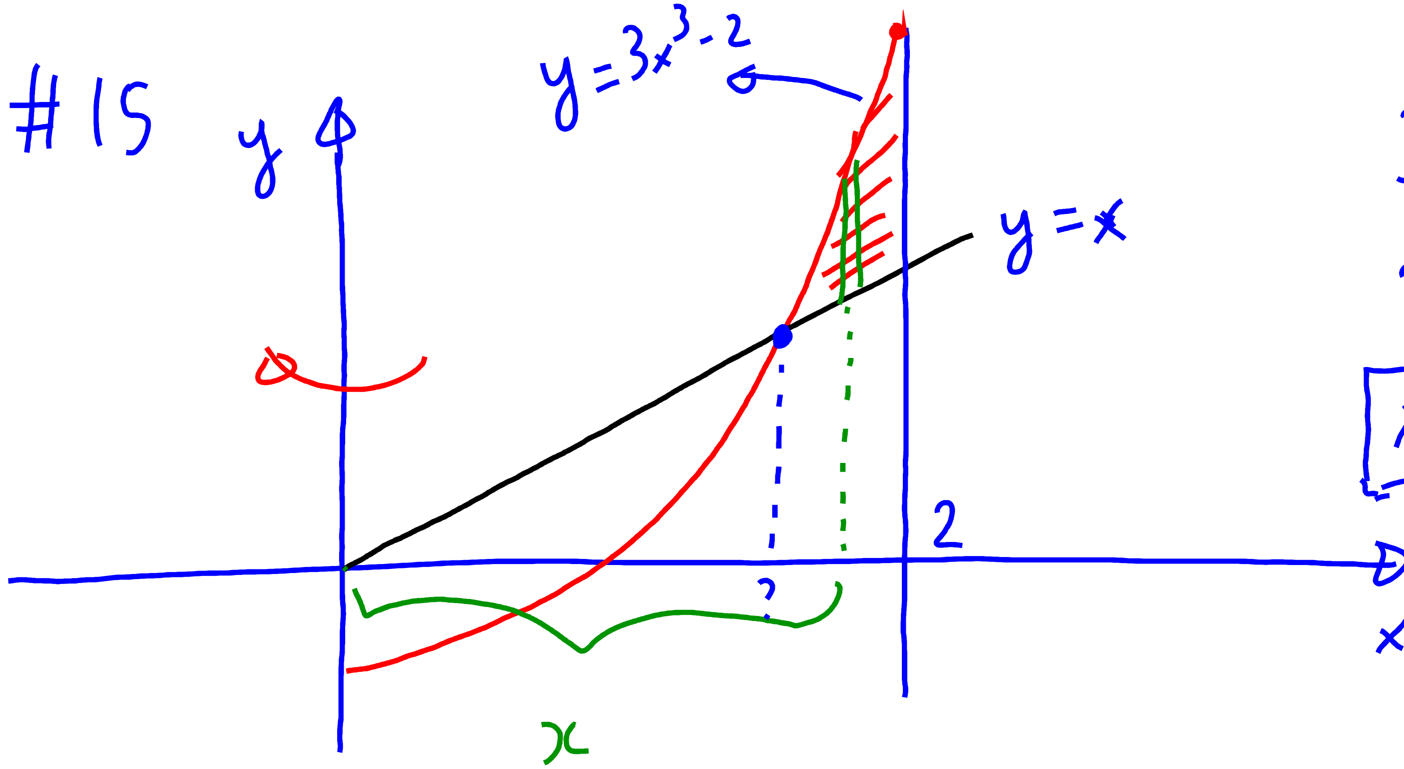


$$2\pi \int_0^2 y \cdot y \cdot e^{y}\, dy$$

$$= 2\pi \cdot \int_0^2 y^2 e^{y}\, dy = \boxed{4\pi(e^2-1)}$$

(Integration by parts - covered later)

#15



$$3x^3 - 2 = x$$

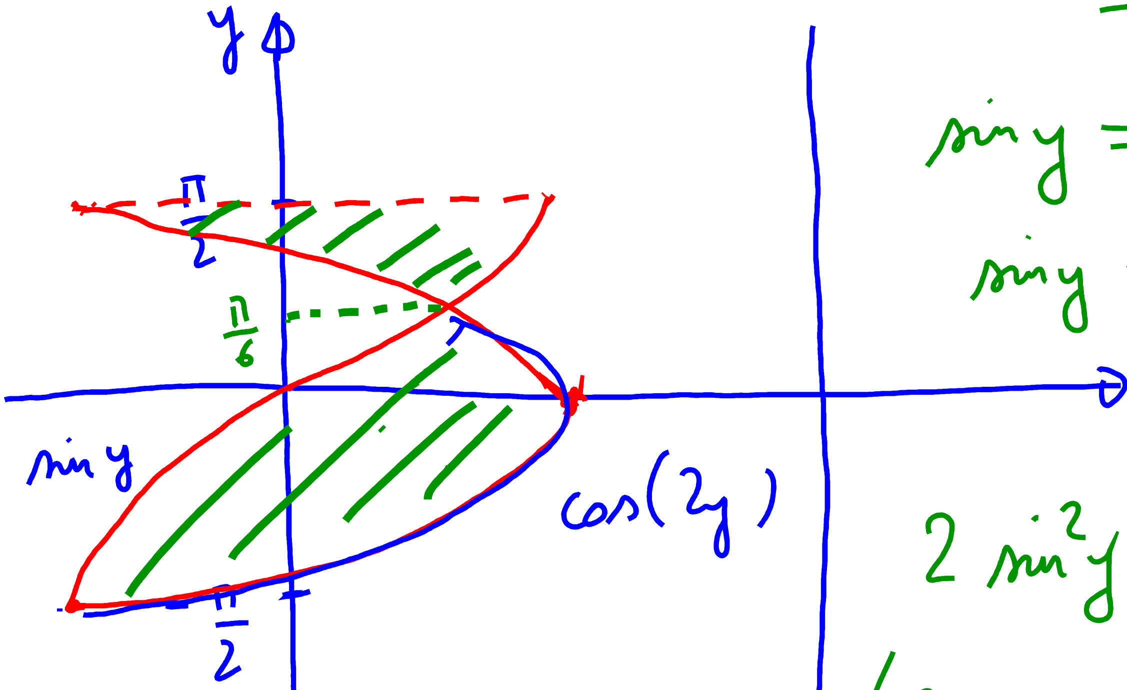
$$3x^3 - x - 2 = 0$$

$\boxed{x = 1}$ will solve the equation

$$2\pi \int_{\boxed{?}}^{2} x \cdot \left(3x^3 - 2 - x\right)\, dx = 2\pi \cdot \int_{1}^{2} x \cdot \left(3x^3 - 2 - x\right) dx$$

1e. Review sheet

$x = \sin y \; ; \quad x = \cos(2y) \; ; \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2} .$



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \left(\cos(2y) - \sin y \right) dy$$

$$\int_{\pi/6}^{\pi/2} \left(\sin y - \cos(2y) \right) dy$$

$\sin y = \cos(2y)$

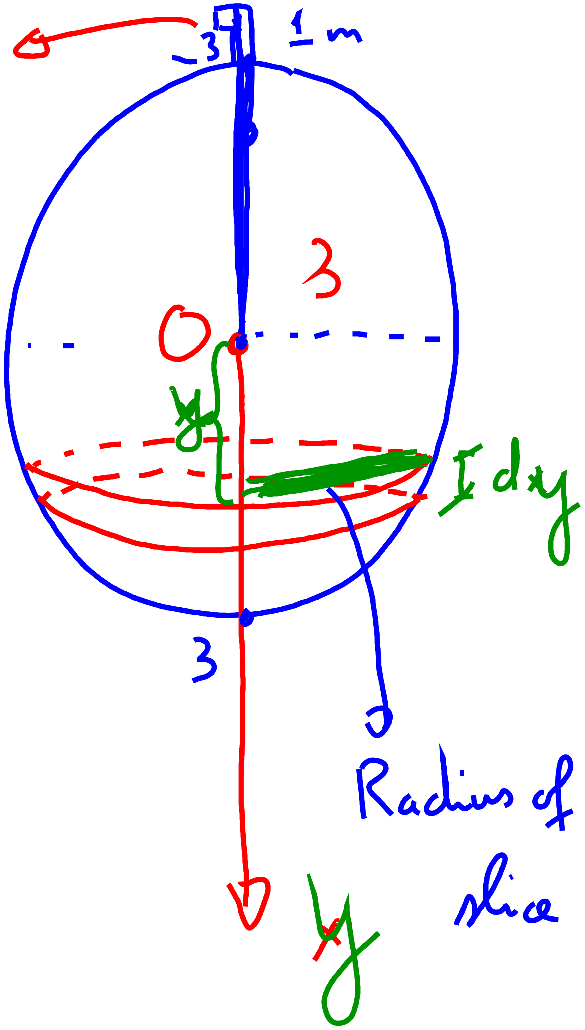
$\sin y = 1 - 2\sin^2 y$

$2\sin^2 y + \sin y - 1 = 0$

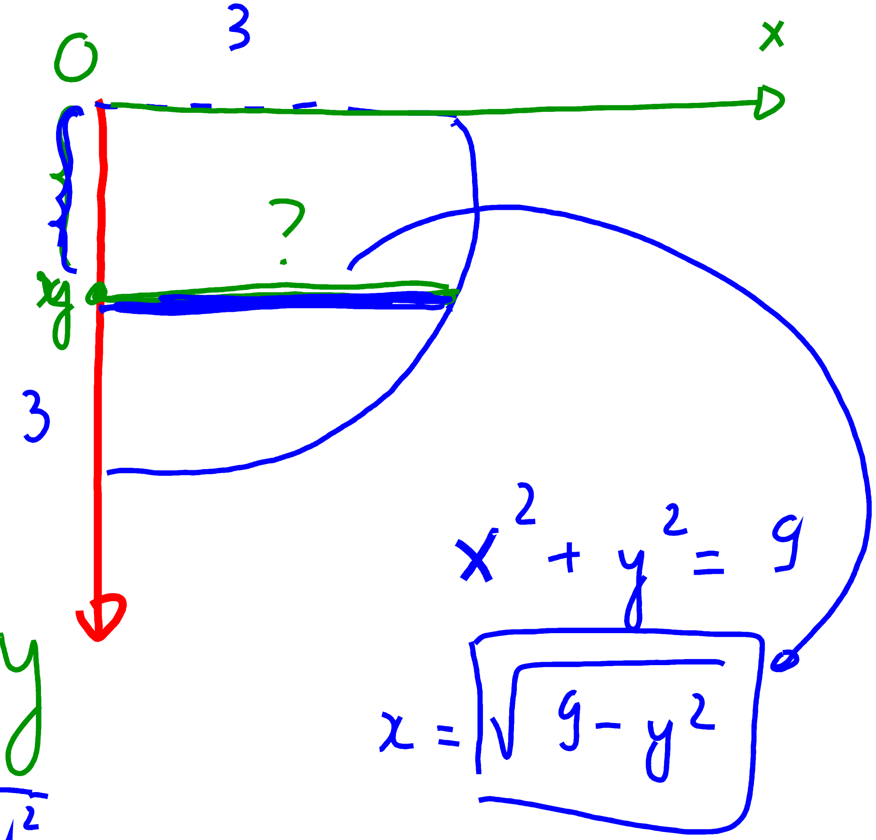
$(2\sin y - 1)(\sin y + 1) = 0$

$\sin y = \frac{1}{2} \; ; \quad \sin y = -1$

$y = \frac{\pi}{6}$



Radius of slice $= \sqrt{9-y^2}$



$$x^2 + y^2 = 9$$

$$x = \boxed{\sqrt{9-y^2}}$$

$$V_{slice} = \pi \cdot (9-y^2) \cdot dy$$

$$m_{slice} = \boxed{1000} \cdot \pi (9-y^2) \cdot dy$$

$$force_{slice} = 1000 \cdot (9.8) \cdot \pi (9-y^2) dy$$

Distance $= 4 + y$

Work$_{slice}$ $= 1000 \cdot (9.8) \cdot \pi (9 - y^2)(4+y)\, dy$

Total work: $= \int_{-3}^{3} 1000 \cdot (9.8) \cdot \pi (9-y^2)(4+y)\, dy$

$= \boxed{1000 \cdot (9.8)\pi} \int_{-3}^{3} (9-y^2)(4+y)\, dy$