2.5. Quadratic Equations

Goals:

1. Solve quadratic equation by
   * Factoring
   * Square Root Property
   * Completing the square
   * Quadratic Formula.
2. Discriminant.
3. Pythagorean Theorem.

Solving quadratic equations by factoring.

Zero Product Property:

If $A \cdot B = 0$, then either $A = 0$ or $B = 0$.

E.g. $x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

By the zero product property, either $x - 7 = 0$ or $x + 3 = 0$.

$\boxed{x = 7}$ $\boxed{x = -3}$

E.g. $x^2 - 25 = 0$

$(x+5)(x-5) = 0$

Either $x + 5 = 0$ or $x - 5 = 0$

$\boxed{x = -5}$; $\boxed{x = 5}$

$\boxed{a}x^2 + \boxed{b}x + \boxed{c} = 0$

E.g. $12x^2 + 11x + 2 = 0$

$12 \cdot 2 = 24 = 8 \cdot 3$

$\underbrace{12x^2 + 3x} + \underbrace{8x + 2} = 0$

$3x(4x+1) + 2(4x+1) = 0$

$(4x+1)(3x+2) = 0$

$4x + 1 = 0$ or $3x + 2 = 0$

$\boxed{x = -1/4}$ $\boxed{x = -2/3}$

② Square Root Property:

If $A^2 = k$, then $A = \sqrt{k}$ or $A = -\sqrt{k}$.

$$(A = \pm\sqrt{k})$$

E.g. $x^2 = 8$

$$\pm\sqrt{8} = \pm\sqrt{4 \cdot 2} = \pm\sqrt{4}\cdot\sqrt{2} = \pm 2\sqrt{2}$$

$$\boxed{x = \pm 2\sqrt{2}}$$

E.g. $4x^2 + 1 = 7$

$$4x^2 = 6$$

$$x^2 = \frac{6}{4} = \frac{3}{2}$$

$$x^2 = \frac{3}{2}$$

$$x = \pm\sqrt{\frac{3}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2}$$

E.g. $3(2x+1)^2 = 15$

$$(2x+1)^2 = 5$$

$$2x + 1 = \pm\sqrt{5}$$

$2x + 1 = \sqrt{5}$ or $2x + 1 = -\sqrt{5}$

$2x = \sqrt{5} - 1$ or $2x = -\sqrt{5} - 1$

$x = \boxed{\frac{\sqrt{5} - 1}{2}}$ or $x = \boxed{\frac{-\sqrt{5} - 1}{2}}$.

3 Completing the Square

$ax^2+bx+c=0$

Solve $x^2+4x+1=0$

Step 1: $x^2+4x = -1$ (Subtract c from both sides)

Step 2: $x^2+4x+4 = -1+4$ (Find $\frac{b}{2}$. $\frac{4}{2}=2$.

$x^2+4x+4 = 3$

Square that $(2)^2=4$

Add $\left(\frac{b}{2}\right)^2$ to both sides)

Step 3: $(x+2)^2 = 3$ (Factor left hand side to complete the square)

Step 4: $x+2=\sqrt{3}$; $x+2=-\sqrt{3}$ (Solve for x using the Square root property)

$x=\boxed{\sqrt{3}-2}$; $x=\boxed{-\sqrt{3}-2}$

E.g. $x^2 - 3x - 5 = 0$

Step 1: $x^2 - 3x = 5$

Step 2: $x^2 - 3x + \frac{9}{4} = 5 + \frac{9}{4}$

$$x^2 - 3x + \frac{9}{4} = \frac{29}{4}$$

Step 3: $\underbrace{\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right)}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

$\cancel{-5 = -1 \cdot 5 = 1 \cdot (-5)}$

Can't factor nicely.

$$\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

Step 4:

$$x - \frac{3}{2} = \sqrt{\frac{29}{4}} \quad \text{or} \quad x - \frac{3}{2} = -\sqrt{\frac{29}{4}}$$

$$x - \frac{3}{2} = \frac{\sqrt{29}}{2} \quad \text{or} \quad x - \frac{3}{2} = -\frac{\sqrt{29}}{2}$$

$$x = \frac{\sqrt{29}}{2} + \frac{3}{2} = \boxed{\frac{\sqrt{29} + 3}{2}}; \quad x = \boxed{\frac{-\sqrt{29} + 3}{2}}$$

4 Quadratic Formula:

$ax^2 + bx + c = 0$; a, b, c : any number
$a \neq 0$.

The quadratic formula says that the two solutions to this equation are

$$x = \frac{-b + \sqrt{\boxed{b^2 - 4ac}}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

In a more compact form : $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

E.g.: * $x^2 + 5x + 1 = 0$; $a = 1$ $b = 5$ $c = 1$

$$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2} = \boxed{\frac{-5 \pm \sqrt{21}}{2}}$$

* Complex numbers :
(Numbers with imaginary part)

E.g. $2 + 3i$

$i^2 = -1$.

$\sqrt{-25}$
$= \sqrt{-1 \cdot 25} = \sqrt{-1}\sqrt{25}$
$= 5i$

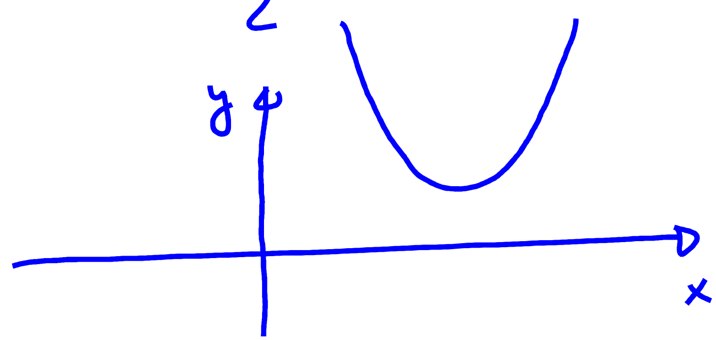
E.g. of a quadratic equation with complex solutions that are not real #.

$$x^2 + x + 2 = 0 \quad ; \quad a = 1 \quad b = 1 \quad c = 2$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$x = \frac{-1 \pm \sqrt{1 - 8}}{2} = \frac{-1 \pm \sqrt{-7}}{2} = \frac{-1 \pm \sqrt{-1} \cdot \sqrt{7}}{2}$$

$$x = \frac{-1 \pm i\sqrt{7}}{2}$$



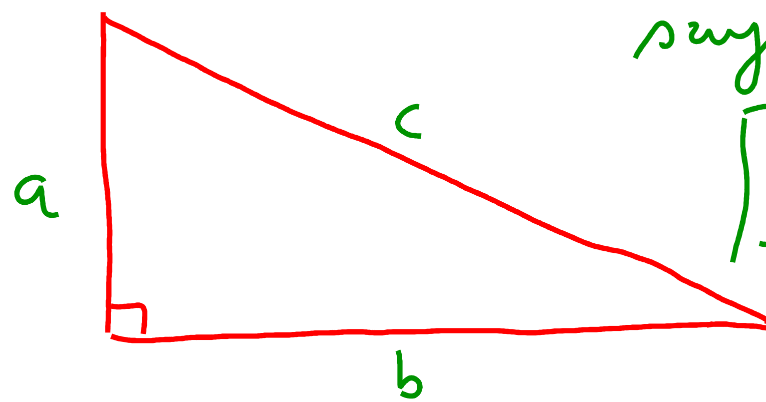
The quantity b^2-4ac is called the discriminant of the equation $ax^2+bx+c=0$

If $b^2-4ac>0$, the equation has 2 real solutions

If $b^2-4ac<0$, the equation has 2 complex solutions that are not real.

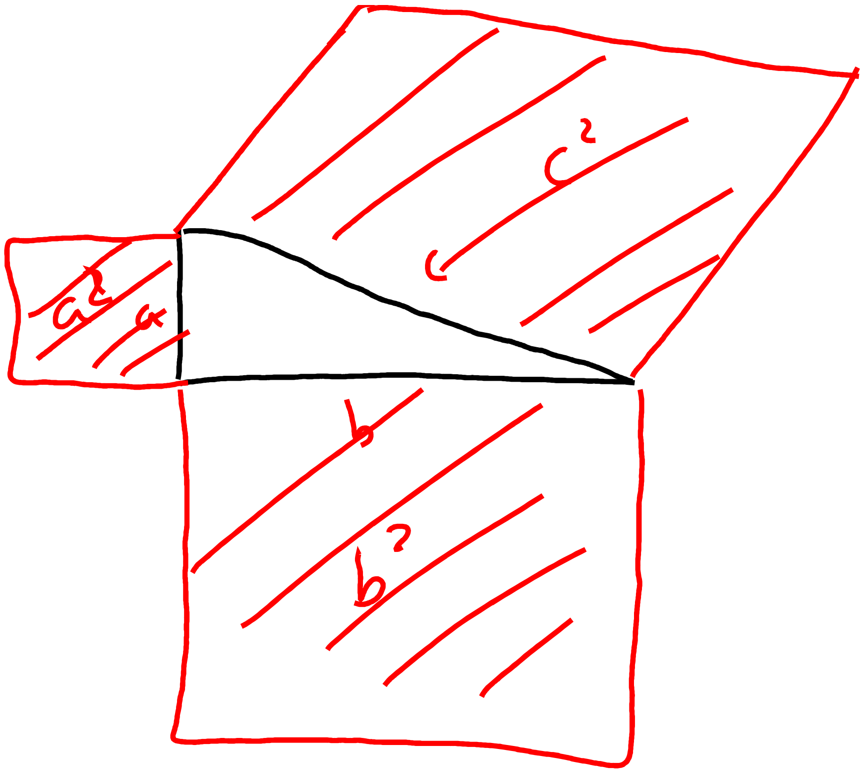
If $b^2-4ac=0$, the equation has exactly 1 real solution.

Pythagorean Theorem

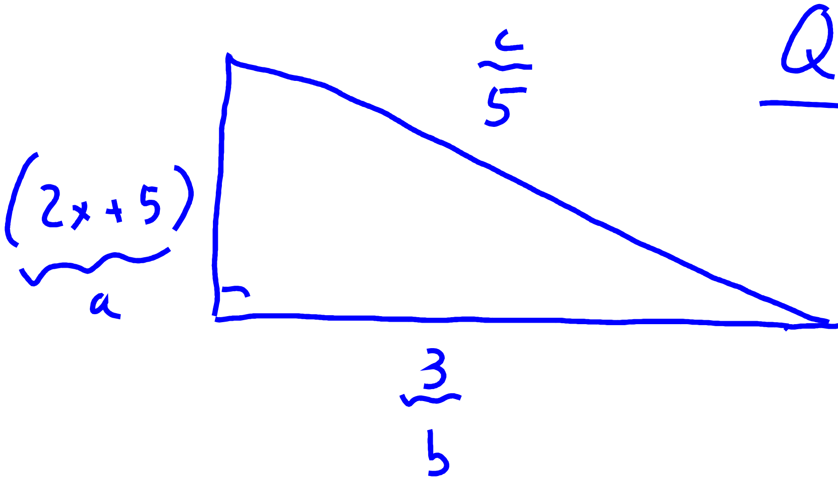


Pythagorean theorem says that

$$\boxed{a^2+b^2=c^2}$$



E.g.



$(4)^2 = 16$; $(-4)^2 = 16$

Question: What must x be

$$(2x+5)^2 + 9 = 25$$

$$(2x+5)^2 = 16$$

$$2x+5 = 4 \quad \text{or} \quad 2x+5 = -4$$

$$x = -\frac{1}{2} \quad ; \quad x = -\frac{9}{2}.$$