2.6. Other types of equations.

Goals:

1. Equations that involve rational exponents.
2. Solve polynomial equations by factoring
3. Solve radical equations / Absolute value equations
4. Equations Quadratic in form
5. Rational equations.

1. Equations with rational exponents.

$(2)^5 = 32$. $(4)^{\frac{1}{2}} = \sqrt{4} = 2$. $(27)^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$(8)^{\frac{2}{3}}$: $\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$; $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$

$$(64)^{-\frac{1}{3}} = \frac{1}{(64)^{1/3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}.$$

In general, we have:

$$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}}$$

Equations:

Solve for x: $x^{\frac{5}{4}} = 32$.

$$\left(x^{\frac{5}{4}}\right)^{\frac{4}{5}} = (32)^{\frac{4}{5}}$$

$$\underbrace{x^{\frac{5}{4}\cdot\frac{4}{5}}}_{x^1 = x}$$

$$x = \left(\sqrt[5]{32}\right)^4$$

$$\boxed{x = 2^4 = 16}$$

Key: Step 1: Raise both sides to an exponent (power) that is the reciprocal of the original one

Step 2: Simplify and solve for x.

E.g. Solve for x: $(x+5)^{\frac{3}{2}} = 8$

$$\left[(x+5)^{\frac{3}{2}}\right]^{\frac{2}{3}} = (8)^{\frac{2}{3}}$$

$$x+5 = 4$$

$$\boxed{x = -1}$$

A slight variation of this type of equation

Solve for x : $$3x^{\frac{3}{4}} = x^{\frac{1}{2}}$$

$$3x^{\frac{3}{4}} = x^{\frac{2}{4}}$$

$$3x^{\frac{3}{4}} - x^{\frac{2}{4}} = 0$$

$$x^{\frac{2}{4}} \cdot \left(3x^{\frac{1}{4}} - 1\right) = 0$$

Either $x^{\frac{2}{4}} = 0$ or $3x^{\frac{1}{4}} - 1 = 0$

$x^{\frac{1}{2}} = 0$; $3x^{\frac{1}{4}} = 1$; $x^{\frac{1}{4}} = \frac{1}{3}$;

$\boxed{x = 0}$; $x = \left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{1}{81}}$

2) Polynomial Equations

E.g. $12x^4 = 3x^2$

$$12x^4 - 3x^2 = 0$$

$$3x^2 \cdot (4x^2 - 1) = 0$$

Either $3x^2 = 0$ or $4x^2 - 1 = 0$

$$x^2 = 0 \qquad 4x^2 = 1$$

$$\boxed{x = 0} \qquad x^2 = \frac{1}{4}$$

$$\boxed{x = \frac{1}{2}} \; ; \; \boxed{x = -\frac{1}{2}}$$

E.g. $\underbrace{x^3 + x^2} \underbrace{- 9x - 9} = 0$

$x^2(x+1) - 9(x+1) = 0$

$(x+1)(x^2-9) = 0$

$(x+1)(x+3)(x-3) = 0$

$\boxed{x=-1}$; $\boxed{x=-3}$; $\boxed{x=3}$

3 Radical Equations

E.g. $\sqrt{x+3} = 3x - 1$. Solve for x

Square both sides: $\left(\sqrt{x+3}\right)^2 = (3x-1)^2$

$x + 3 = (3x-1)(3x-1)$

$x + 3 = 9x^2 - 6x + 1$

$0 = 9x^2 - 7x - 2$

$0 = 9x^2 - 9x + 2x - 2$

$ac = -18$
$= -9 \cdot 2$

$9x(x-1) + 2(x-1) = 0$

$(9x+2)(x-1) = 0$

$x = \boxed{-\frac{2}{9}}$; $x = \boxed{1}$

Check to eliminate extraneous solutions

Check $\boxed{\cancel{x = -\frac{2}{9}}}$.

$$\sqrt{\boxed{-\frac{2}{9}+3}} = 3\cdot\left(-\frac{2}{9}\right) - 1$$

$$\underbrace{\sqrt{\frac{25}{9}}}_{\frac{5}{3}} = \underbrace{-\frac{6}{9} - 1}_{-\frac{15}{9} = -\frac{5}{3}}$$

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So, $x = -\frac{2}{9}$ is an extraneous solution

Check $\boxed{x = 1}$ ✓

$$\sqrt{1+3} = 3\cdot 1 - 1$$

$$\sqrt{4} = 2$$

E.g.

$$\boxed{\sqrt{3x+7} + \sqrt{x+2} = 1}$$

$$\sqrt{3x+7} = 1 - \sqrt{x+2}$$

$$3x+7 = \left(1-\sqrt{x+2}\right)^2$$

$$\left(1-\sqrt{x+2}\right)\left(1-\sqrt{x+2}\right)$$

$$3y+7 = 1 - 2\sqrt{x+2} + x+2$$

$$3x+7 = 3 - 2\sqrt{x+2} + x$$

$$2x+4 = -2\sqrt{x+2}$$

$$-x-2 = \sqrt{x+2}$$

$$(-x-2)^2 = x+2$$

$$(-x-2)(-x-2)$$

$$= x+2$$

$$x^2+4x+4$$

$$= x+2$$

$$x^2+3x+2=0$$

$$(x+1)(x+2)=0$$

$$\cancel{\boxed{x=-1}} \text{ or } \boxed{x=-2}$$

Key in solving radical equations

1. Square both sides to get rid of the radical.

2. Check the solutions for extraneous solutions.

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4 Absolute Value Equations

<u>Key</u>: $|\text{Stuff}| = c$ → a #

<u>Case 1</u>: If c is a positive #, either $\text{Stuff} = c$
or $\text{Stuff} = -c$

E.g. $|x| = 8$ then either $x = 8$ or $x = -8$

<u>Case 2</u>: If $c = 0$, then $\text{Stuff} = 0$

<u>Case 3</u>: If $c < 0$, No Solution.

E.g. $|6x+4| = 8$; $|3x+4| = -9$; $|3x-5| - 4 = 6$

$|-5x+10| = 0$

* $|6x+4| = 8$; $6x+4=8$ or $6x+4=-8$

$\boxed{x = \frac{2}{3}}$; $\boxed{x = -2}$

* $|3x+4| = -9$. No Solutions

* $|3x-5| - 4 = 6$; $|3x-5| = 10$

$3x-5 = 10$ or $3x-5 = -10$

$\boxed{x = 5}$ $\boxed{x = -\frac{5}{3}}$

$|-5x+10| = 0$

$-5x+10 = 0$

$-5x = -10$

$\boxed{x = 2}$

(5) Equations Quadratic in form. (Substitution method)

E.g. $(\boxed{x^2 - 1})^2 + (\boxed{x^2 - 1}) - 12 = 0$

let $A = x^2 - 1$

$$A^2 + A - 12 = 0$$

$$(A-3)(A+4) = 0$$

$A = 3$ or $A = -4$

$x^2 - 1 = 3$ or $x^2 - 1 = -4$

$x^2 = 4$ $\qquad$ $x^2 = -3$

$x = \boxed{\pm 2}$ $\qquad$ $x = \boxed{\pm i\sqrt{3}}$.

6 Rational Equations :

$$\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$$

$$\frac{3x+2}{x-2} + \frac{1}{x} = \boxed{\frac{-2}{x(x-2)}}$$

GCD : $(x-2)x$

Multiply both sides by $(x-2)x$

$$\cancel{(x-2)}x \cdot \frac{3x+2}{\cancel{x-2}} + (x-2)\cancel{x} \cdot \frac{1}{\cancel{x}} = \cancel{(x-2)x} \cdot \frac{-2}{\cancel{x(x-2)}}$$

$$x \cdot (3x+2) + x - 2 = -2$$

$$3x^2 + 2x + x \cancel{-2} \cancel{+2} = 0$$

$$3x^2 + 3x = 0 \; ; \quad 3x(x+1) = 0 \; ; \quad \cancel{x = 0} \text{ or } \boxed{x = -1}$$