3.1. Functions and Function Notations

Goals:

1. Understand the concepts of a relation and a function.
2. Evaluate functions.
3. Determine whether a function is one-to-one
4. Use vertical line test to identify functions
5. Know the graphs of basic functions

1) Relations and Functions

* A relation is a set of ordered pairs.

set → collection of objects

E.g. $\{(1,2);(2,4);(3,6);(4,8);(5,10)\}$

call this relation R1.

E.g. $\{(\text{odd},1);(\text{even},2);(\text{odd},3);(\text{even},4)\} \longrightarrow$ R2

E.g. {(plain donut, $1.49); (Jelly Donut, $1.99); (Chocolate donut, $1.99)} ⟶ R3

Given a relation, the domain is the set consisting of the first components of each ordered pair.

The range is the set consisting of the second components of each ordered pair.

E.g. For R1. Domain = $\{1, 2, 3, 4, 5\}$

Range = $\{2, 4, 6, 8, 10\}$

For R2. Domain = {odd, even}

Range = $\{1, 2, 3, 4\}$

For R3. Domain = {Plain Donut, Jelly Donut, Chocolate Donut}

Range = {\$1.49, \$1.99}.

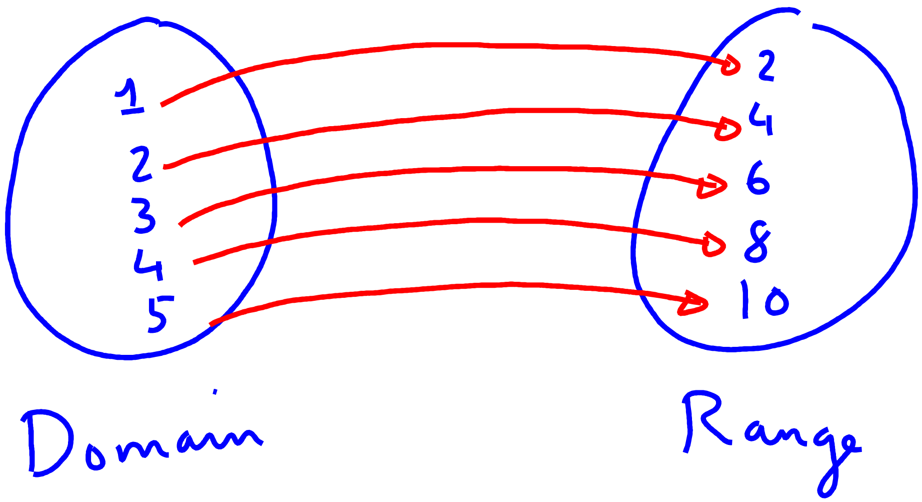
Each value in the domain is usually called an input or an independent variable of the relation and is denoted by the letter x. (x = plain donut.)

Each value in the range is called an output or a dependent variable, denoted by y.

What is a function?

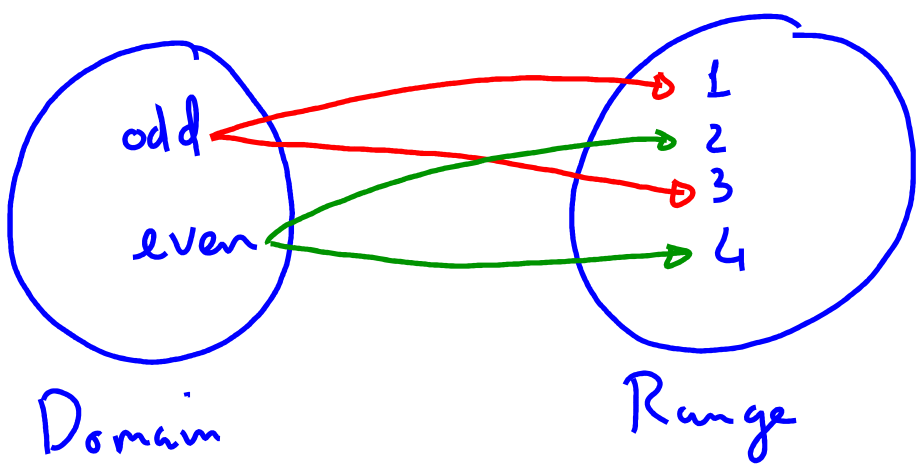
A function is a relation that asigns a single output in the range to each input in the domain.

E.g. $R1 = \{(1,2),(2,4);(3,6);(4,8);(5,10)\}$



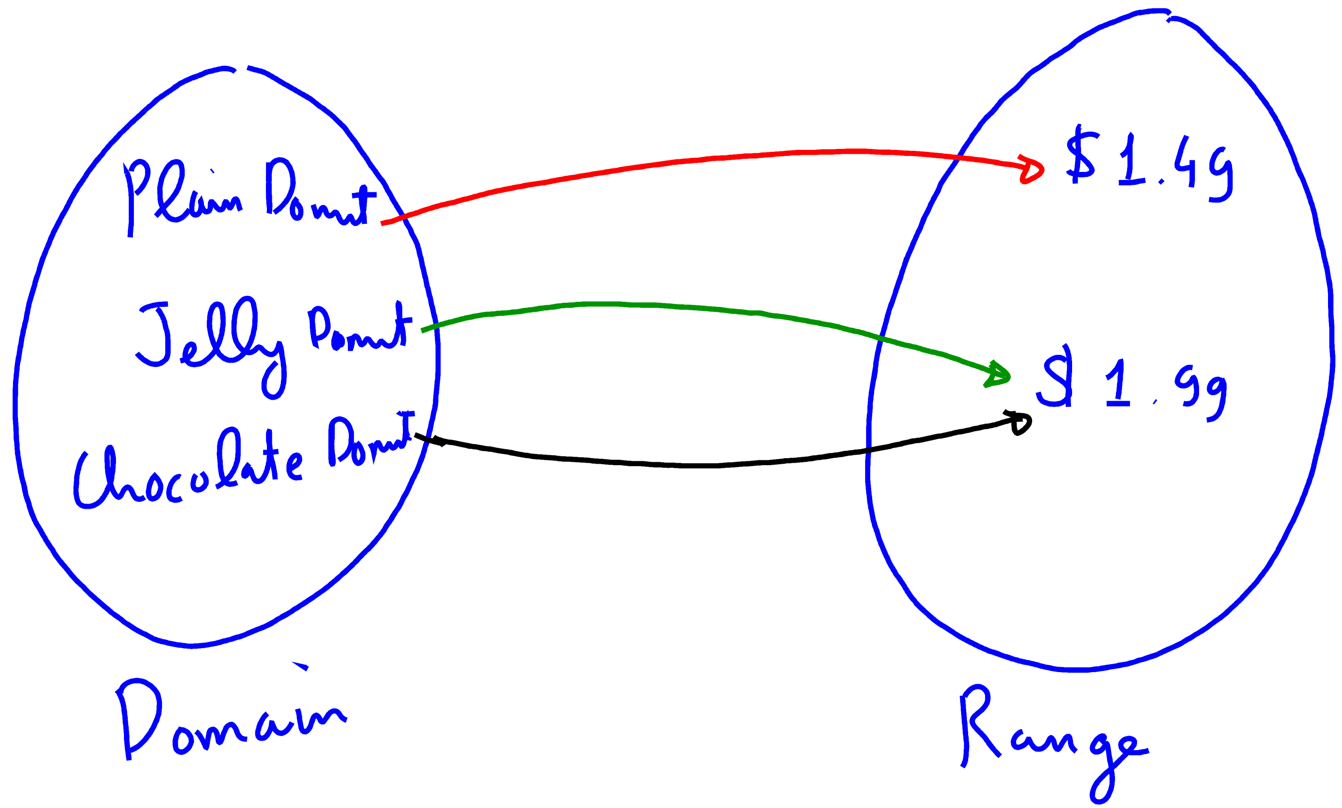
This relation is a function

R2.



This relation is NOT a function.

R3



This is a function.

Function: $\underline{R1}$ is a function. Let's name this function f.

$$R1 = \{(1,2);(2,4);(3,6);(4,8);(5,10)\}$$

$$f(\boxed{1}) = \boxed{2}$$

output

name of the function

input

Read this as: f of 1 is equal to 2 or the value of the function at $x = 1$ is 2.

R3: let's name this function a

R3 = { (plain donut, \$1.49); (jelly donut, \$1.99);
(chocolate donut, \$1.99) }

a(chocolate donut) = \$1.99.

In general, the notation $y = f(x)$ defines a function named f. This is read as y is a function of x.

② Evaluate functions

When you have a formula: $f(x) = \boxed{x^2 + 3x - 4}$, this defines a function. The set of ordered pairs for this function is the set consisting of $\left(\boxed{x}, \boxed{x^2 + 3x - 4}\right)$

E.g. $\left(x, \boxed{x^2+3x-4}\right)$

$(4)^2 + 3\cdot(4) - 4 = 16 + 12 - 4$
$= 24$.

$x = 4$

$\underbrace{(4, 24)}$; $\underbrace{\left(\frac{2}{3}, -\frac{14}{9}\right)}$

$f(4) = 24$

$\left(\frac{2}{3}\right)^2 + 3\cdot\left(\frac{2}{3}\right) - 4$

$\frac{4}{9} + 2 - 4 = \frac{4}{9} - 2 = \frac{4-18}{9}$
$= -\frac{14}{9}$.

$f\left(\frac{2}{3}\right) = -\frac{14}{9}$.

When we plug a particular input value into the given formula to calculate the output, we evaluate the function at that input.

Evaluate functions given by a formula.

1. Replace the input variable in the formula with the value provided
2. Calculate the result and simplify.

E.g. $f(x) = x^2 + 3x - 4$

$$f(2) = (2)^2 + 3(2) - 4 = 4 + 6 - 4 = 6.$$

$$f(b) = b^2 + 3b - 4$$

$$f(b+w) = (b+w)^2 + 3(b+w) - 4$$
$$= (b+w)(b+w) + 3b + 3w - 4$$
$$= b^2 + 2bw + w^2 + 3b + 3w - 4$$

$$f(\boxed{7x}) = (7x)^2 + 3\cdot(7x) - 4 = 49x^2 + 21x - 4.$$

$$f(x) = \underline{x^2 + 3x - 4}.$$

Evaluate and simplify: $\dfrac{\boxed{f(a+h)} - \boxed{f(a)}}{h}$.

$f(a+h) =$

$f(a) =$

$$\frac{(a+h)^2 + 3(a+h) - 4 - \left(a^2 + 3a - 4\right)}{h}.$$

$$= \frac{(a+h)(a+h) + 3a + 3h - 4 - a^2 - 3a + 4}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 \cancel{+3a} + 3h \cancel{-4} \cancel{-a^2} \cancel{-3a} \cancel{+4}}{h}$$

$$= \frac{h^2 + 2ah + 3h}{h} = \frac{\cancel{h}(h + 2a + 3)}{\cancel{h}} = h + 2a + 3$$

E.g. Given $h(p) = p^2 + 2p$. $\cancel{h(3)}$

Solve for $\boxed{h(p)} = 3$.

$$p^2 + 2p = 3$$

$$p^2 + 2p - 3 = 0$$

$$(p+3)(p-1) = 0$$

$$p = -3 \; ; \; p = 1$$

There are 2 inputs that give us the output 3 and those inputs are $p = -3$ and $p = 1$

A function can be given by a graph instead of a formula.

Read function values from a given graph.

We did an example in the book.

To evaluate $g(2)$; we locate the point on the graph where $x = 2$; then we read the y-coordinate of that point.

To solve $g(x) = 4$; we locate the point(s) on the graph where $y = 4$; then we read the x-coordinates of those pts.

Vertical Line Test

If we can draw a vertical line that intersects a graph more than once, then the graph is <u>not</u> a graph of a function.