3.2. Domain and Range

Goals:
1. Domain of functions given by formula
2. Domain and range of functions given by graphs
3. Piecewise functions

Domain of a function given by a formula.

If a function is given by a formula $y = f(x)$, the domain is the set of x for which $f(x)$ makes sense, i.e., we can calculate $f(x)$.

Domain of polynomial function

E.g. $f(x) = x^2 + x$

Domain is the set of real numbers

Interval notation: $(-\infty, \infty)$

$D = (-\infty, \infty)$

$g(x) = 2x^5 - 3x^4 + \frac{1}{2}x^3 - \pi x^2 + 0.5x - 0.75$

$D = (-\infty, \infty)$

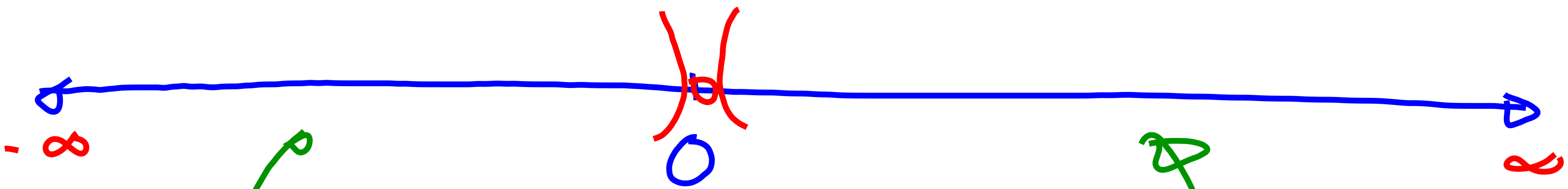
Domain of rational functions:

E.g. $f(x) = \frac{1}{x}$ Domain

$f(7) = \frac{1}{7}$; $f\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = 2$; $\cancel{f(0) = \frac{1}{0}}$

Domain: set of real numbers except for 0

$$D = (-\infty, 0) \cup (0, \infty)$$



$$g(x) = \frac{x+2}{x-3} \qquad D = (-\infty, 3) \cup (3, \infty)$$

Strategy to find the domain of any rational function.

$$f(x) = \frac{\text{Stuff } 1}{\text{Stuff } 2}$$

1 Set denominator Stuff $2 = 0$. Solve for x in that equation.

2 Domain is the set of all real numbers except for the numbers you get from step 1.

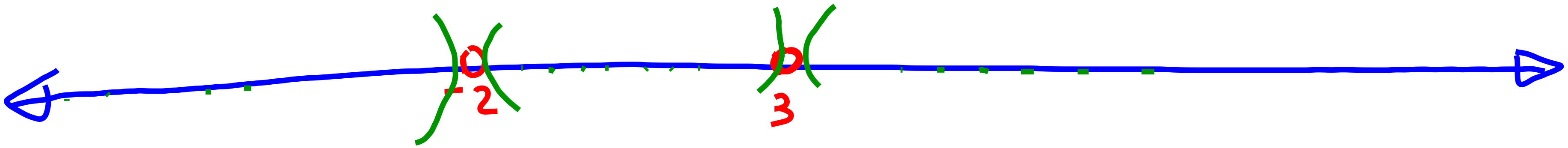
E.g. $f(x) = \dfrac{1}{x^2 - x - 6}$. Find the domain of this function.

① $x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2 \ ; \ x = 3$

② $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$



E.g $g(x) = \dfrac{x+2}{x^2 - x - 8}$

$x^2 - x - 8 = 0$ $\qquad -8 = -1 \cdot 8 = -2 \cdot 4 = 8 \cdot (-1) = 4(-2)$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-8)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1 + 32}}{2} = \boxed{\frac{1 \pm \sqrt{33}}{2}}$$

$$D = \left(-\infty, \frac{1-\sqrt{33}}{2}\right) \cup \left(\frac{1-\sqrt{33}}{2}, \frac{1+\sqrt{33}}{2}\right) \cup \left(\frac{1+\sqrt{33}}{2}, \infty\right)$$

$h(x) = \dfrac{2x}{x^2+4}$. Find the domain of h.

$D = (-\infty, \infty)$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Domains of radical functions.

$f(x) = \sqrt{x}$.

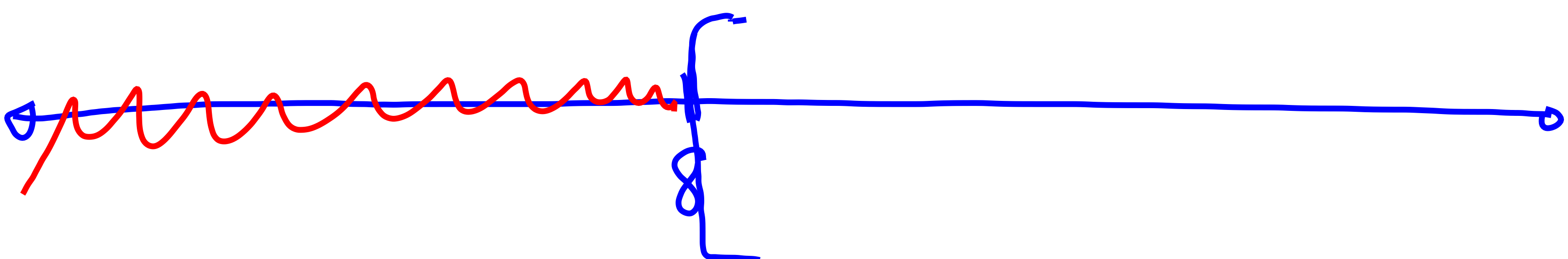
Domain: the set of all non-negative numbers.

$D = [0, \infty)$

$g(x) = \sqrt{x - 8}$

Domain = set of all real numbers that are not less than 8

$D = [8, \infty)$



Strategy to find the domain of functions with square root.

$$f(x) = \sqrt{\text{Stuff}}$$

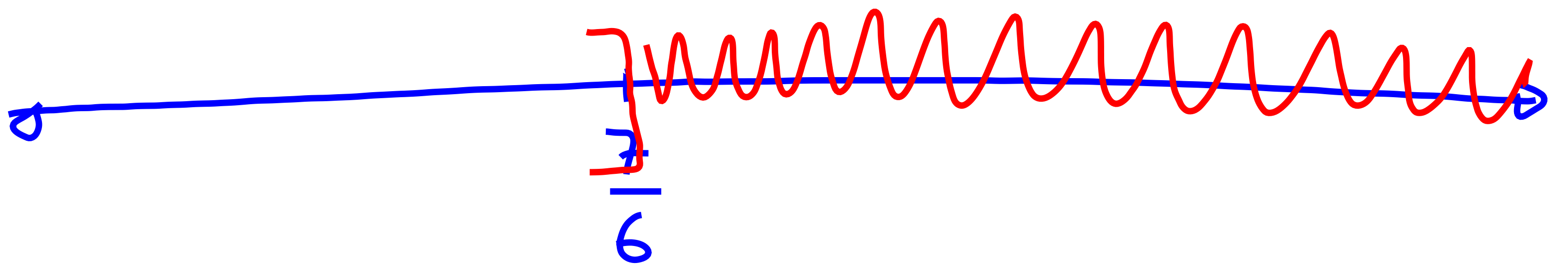
1. Solve the inequality Stuff ≥ 0
2. the solutions to the above inequality is the domain.

E.g. $g(x) = \sqrt{\boxed{7-6x}}$ Find the domain of g.

(1) $7 - 6x \geq 0$

$$\frac{-6x}{-6} \geq \frac{-7}{-6} \; ; \quad \boxed{x \leq \frac{7}{6}}$$

(2) Domain: $\left(-\infty, \frac{7}{6}\right]$

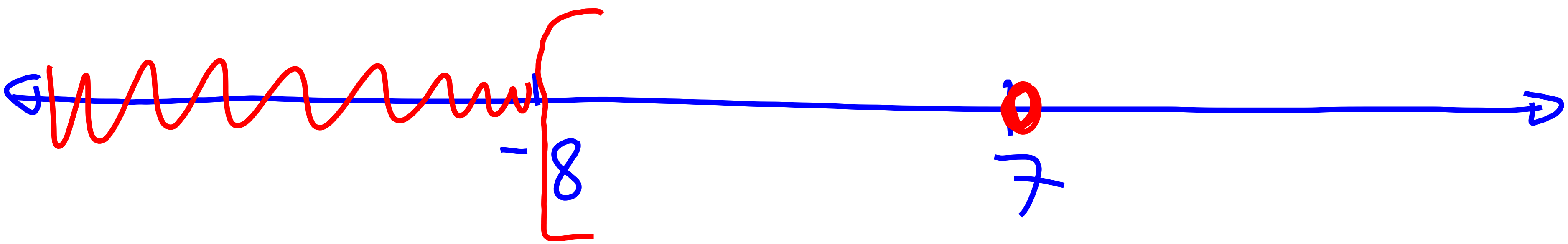


Mixed functions:

E.g. $g(x) = \frac{\sqrt{x+8}}{x-7}$. Find the domain of this function.

* x cannot be 7.

* $x + 8 \geq 0$; $x \geq -8$



$D = [-8, 7) \cup (7, \infty)$

E.g. $h(x) = \dfrac{2x}{\sqrt{5-2x}}$. Find the domain of h.

$5 - 2x > 0$; $\dfrac{-2x}{-2} > \dfrac{-5}{-2}$; $x < \dfrac{5}{2}$

$D = \left(-\infty, \dfrac{5}{2}\right)$

Domain and Range of a function given by a graph.

Domain: set of x

Range. set of y.

Piecewise - functions

E.g. $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \\ x & \text{if } x > 2 \end{cases}$

$f(-4) = (-4)^2 = 16$

$f(0) = (0)^2 = 0$

$f(1) = (1)^2 = 1$

$f(5) = 5$

$f(2017) = 2017$

$f\left(\frac{3}{2}\right) = 3$; $f(1.99999) = 3$