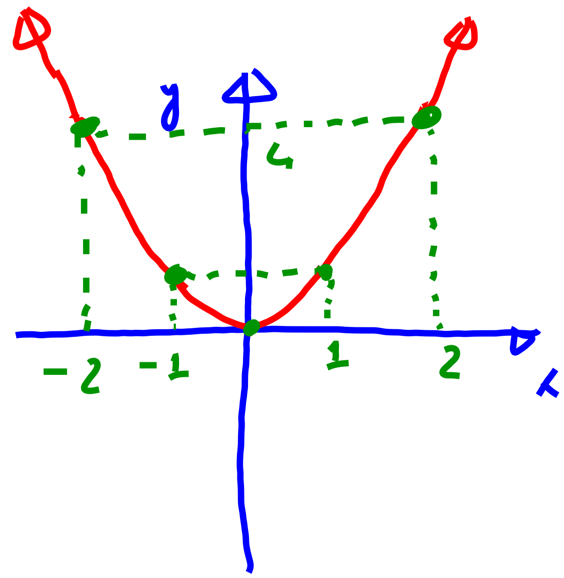
3.5. Transformation of Functions

Goal. Study all the ways to transform (move or resize) the graphs of functions.

Start with $y = f(x) = x^2$ to illustrate the transformations.

Key points on the graph of $y = f(x) = x^2$

x	$y = f(x) = x^2$
0	0
1	1
-1	1
2	4
-2	4

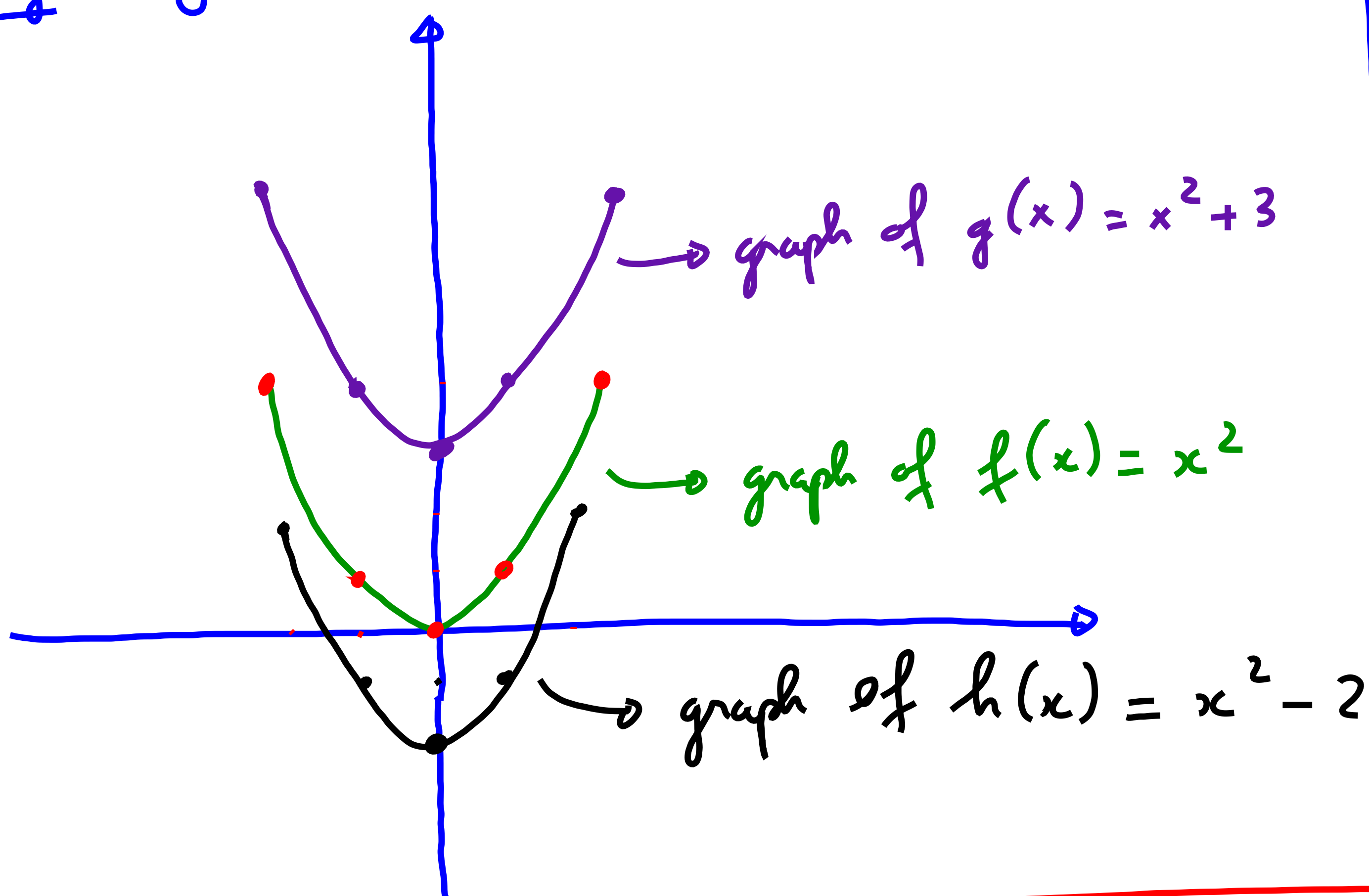


We can move this graph up or down by adding a constant to the y-value

$$g(x) = f(x) + c \; ; \quad c > 0 \; : \text{ move up.}$$
$$g(x) = f(x) + c \; ; \quad c < 0 \; : \text{ move down}$$

E.g. $g(x) = f(x) + 3 = x^2 + 3$

$h(x) = f(x) - 2$
$= x^2 - 2.$



We can move the graph of f left or right by adding a constant to the x-value.

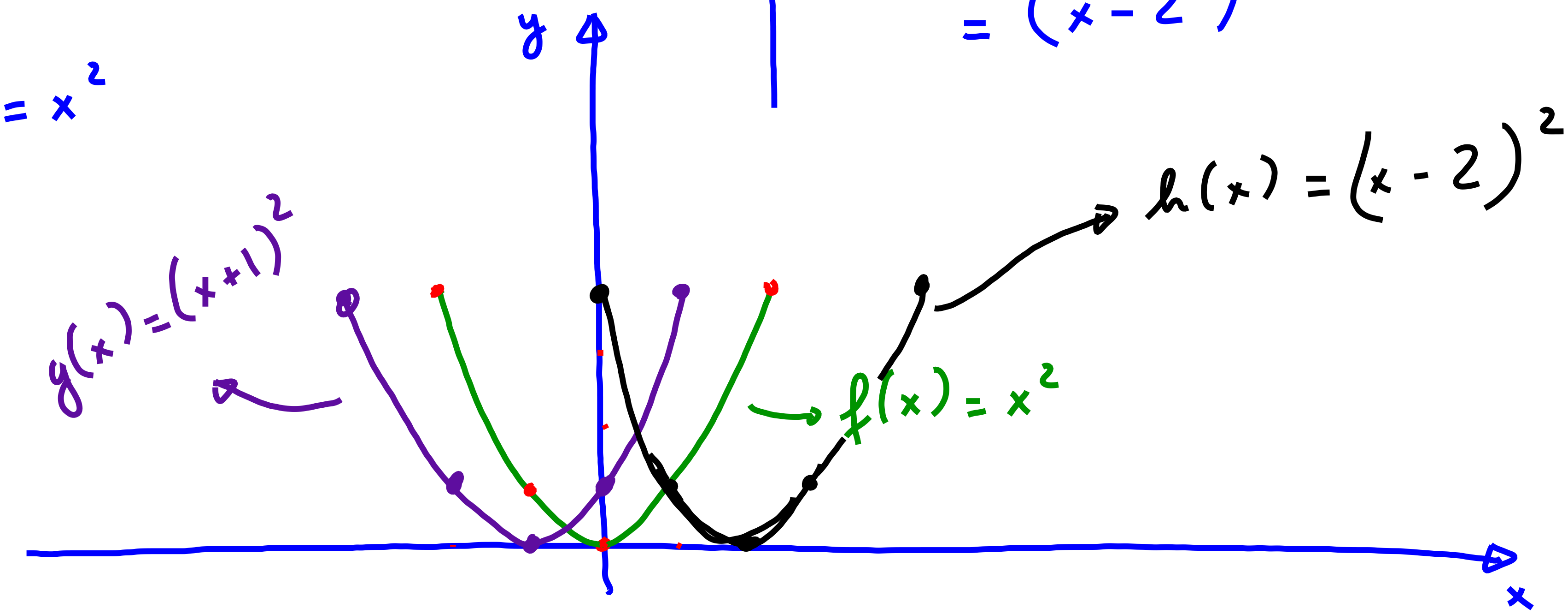
$g(x) = f(x+c)$; $c > 0$: move to the left

$g(x) = f(x+c)$; $c < 0$: move to the right

$g(x) = f(x+1) = (x+1)^2$

$h(x) = f(x-2)$
$= (x-2)^2$

$f(x) = x^2$

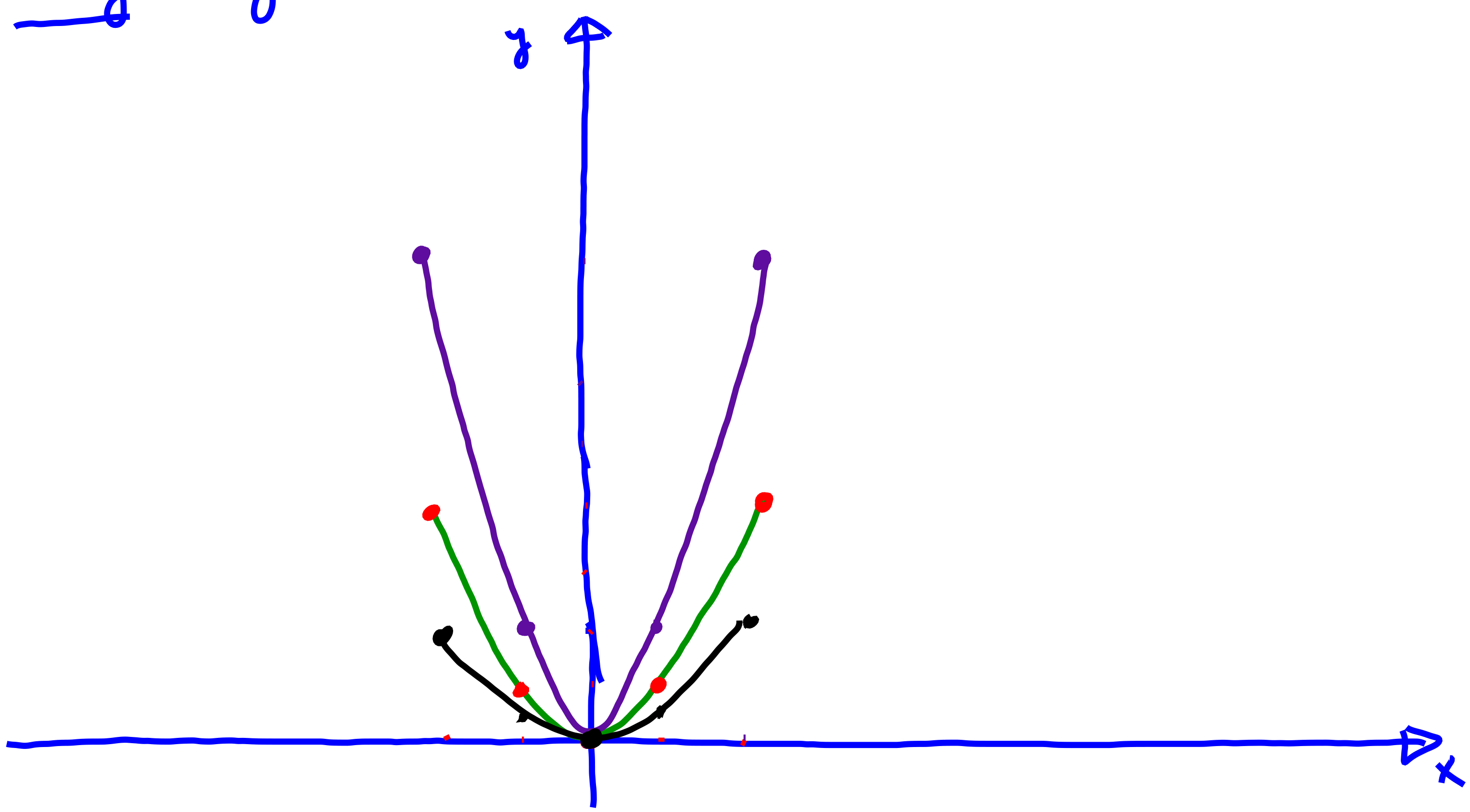


We can stretch or compress the graph of f in the y-direction by multiplying the whole function by a constant.

$g(x) = c f(x)$; $c > 1$: stretch it.

$g(x) = c f(x)$; $0 < c < 1$: compress it.

E.g. $g(x) = 2 f(x) = 2x^2$; $h(x) = \frac{1}{2} f(x) = \frac{1}{2} x^2$



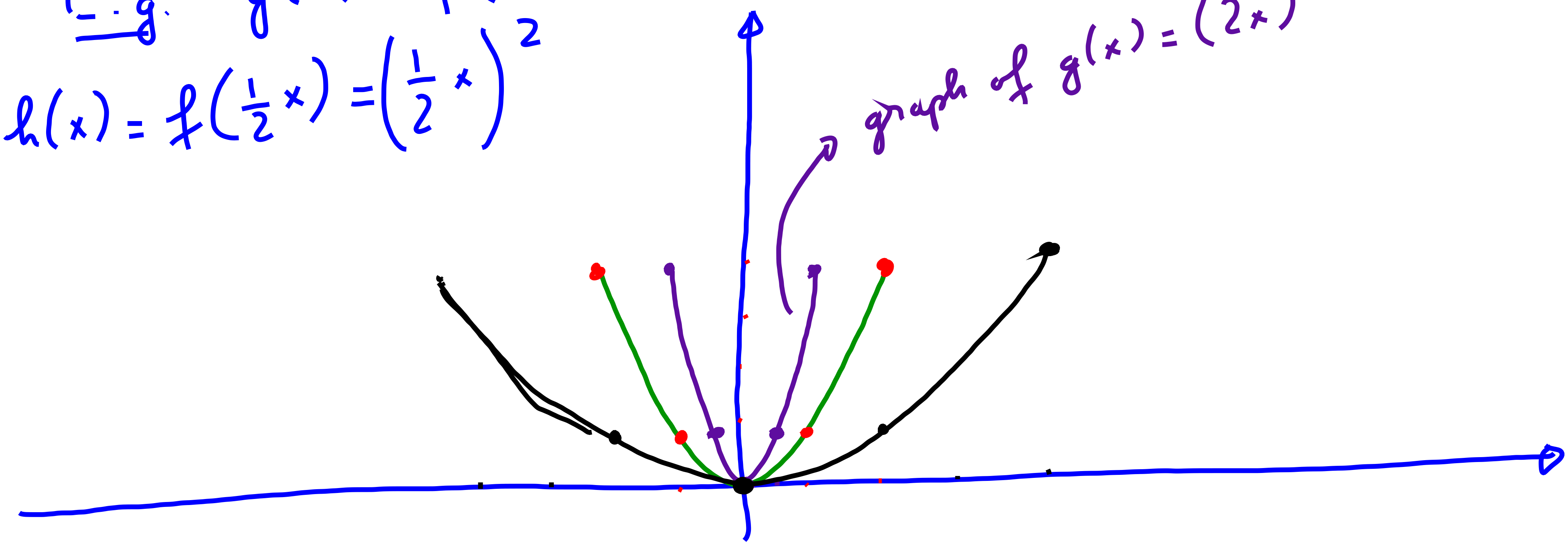
We can stretch or compress the graph of f in the x-direction by multiplying a constant to the x-part

$g(x) = f(cx)$; $c > 1$: compress

$g(x) = f(cx)$; $0 < c < 1$: stretch.

E.g. $g(x) = f(2x) = (2x)^2$

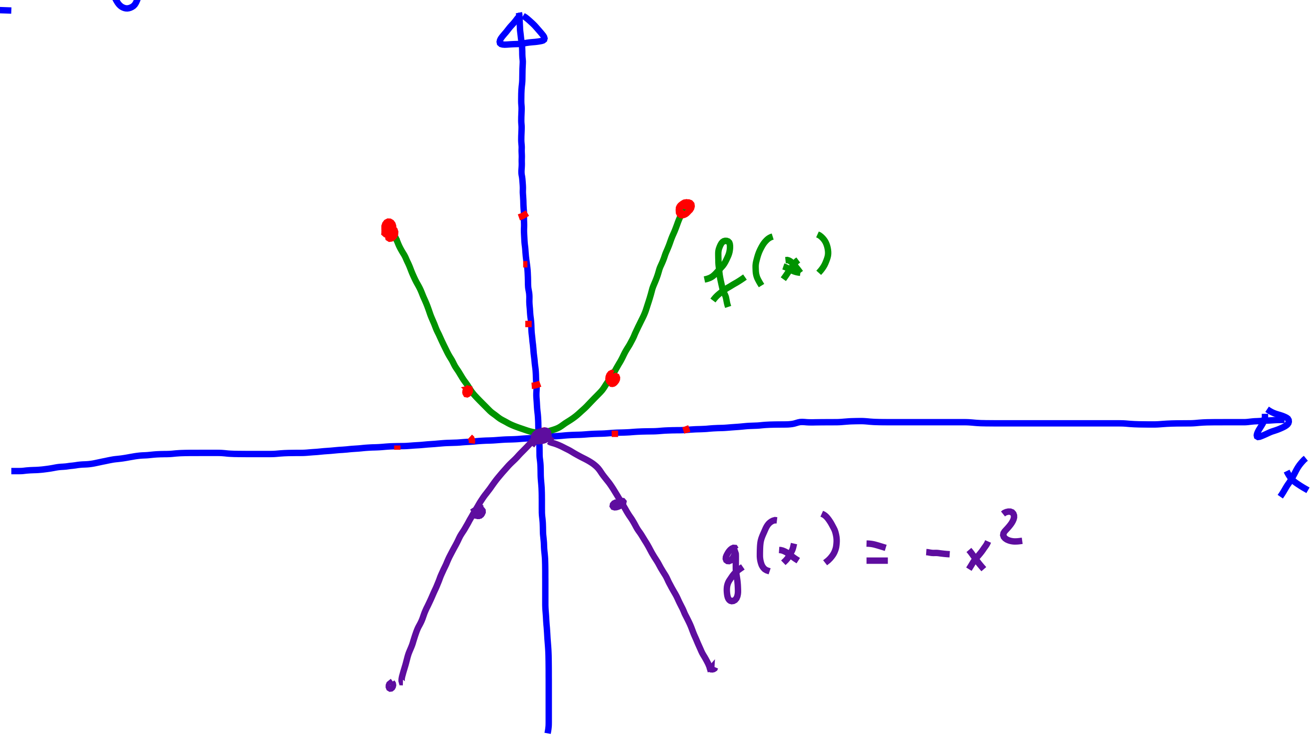
$h(x) = f(\frac{1}{2}x) = \left(\frac{1}{2}x\right)^2$



We can flip the graph of f upside down (across the x-axis) by multiplying the whole function by -1.

$g(x) = -f(x)$: flip across x-axis.

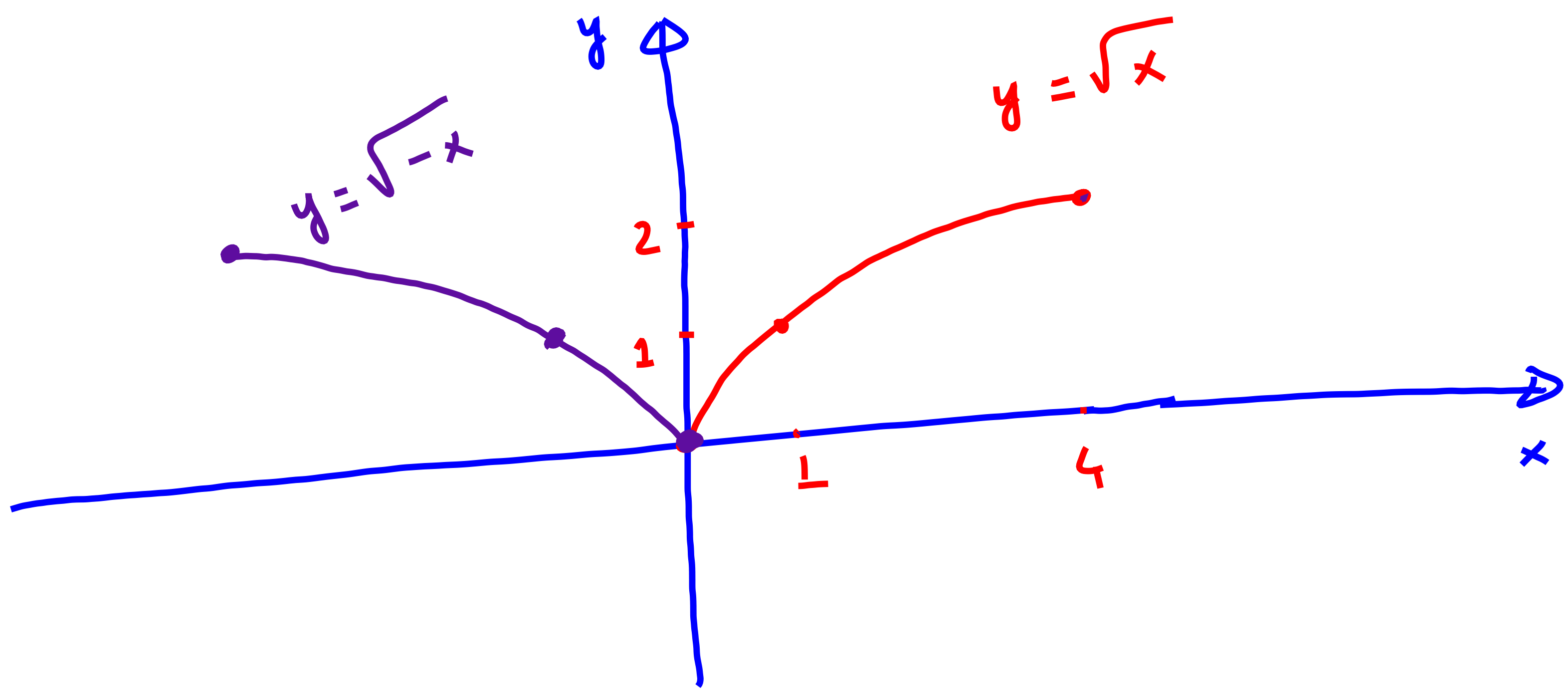
E.g. $g(x) = -x^2$.



We can flip the function f across the y-axis by multiplying the x values by -1

$g(x) = f(-x)$: flip across the y-axis.

$f(x) = \sqrt{x}$; $g(x) = f(-x)$



Summary : $y = f(x)$ ← original function.

$g(x) = f(x) + c$
- $c > 0$: move up
- $c < 0$: move down

$g(x) = f(x+c)$
- $c > 0$: move left
- $c < 0$: move right.

$g(x) = c\,f(x)$
- $c > 1$: stretch in y-direction
- $0 < c < 1$: compress in y-direction

$g(x) = f(cx)$
- $c > 1$: compress in x-direction
- $0 < c < 1$: stretch in x-direction.

$g(x) = -f(x)$: flip across x-axis.

$g(x) = f(-x)$: flip across y-axis.