3.6 - Absolute Value Functions

3.7 - Inverse Functions

Goals: (1) Determine whether 2 given functions are inverses of one another.

(2) Find the inverse function of a one-to-one function.

(3) Evaluate inverse functions and find the domain and range of the inverse function

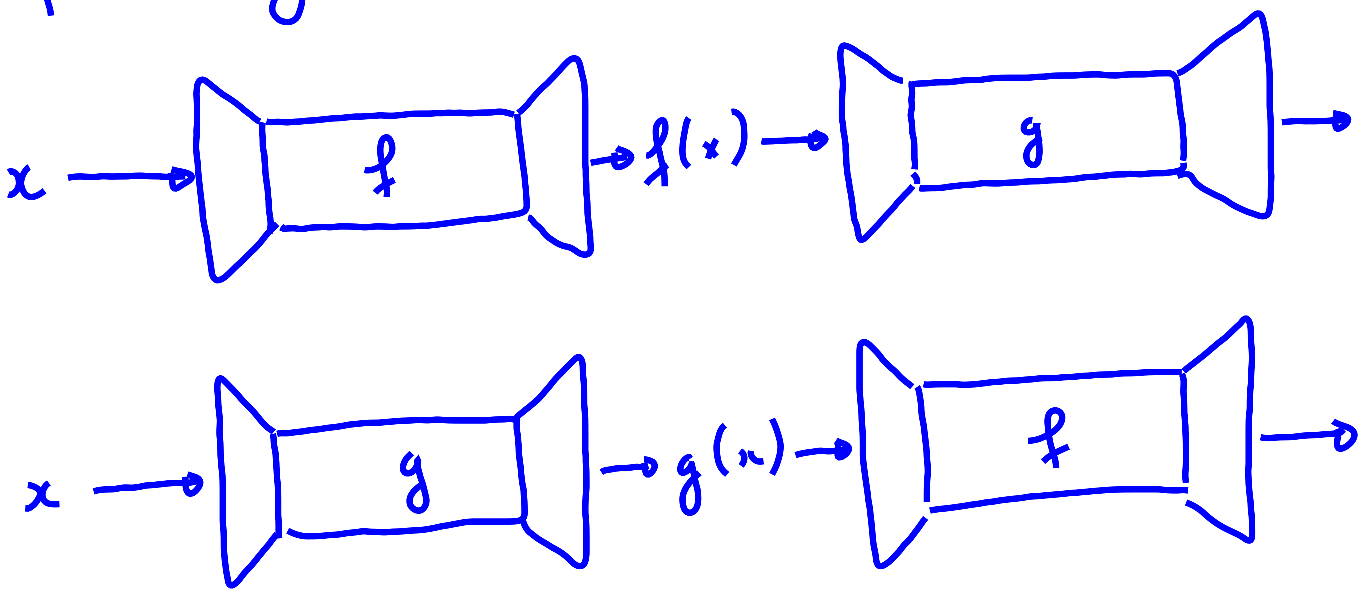
(4) Graphs of Inverse Functions

3.6. * Graph absolute value function.

* Review how to solve absolute value inequalities.

What does it mean for 2 functions to be inverses of one another?

f and g : functions



$$\underbrace{g(f(x)) = x}_{g \text{ undoes what } f \text{ does}}$$

$$\underbrace{f(g(x)) = x}_{f \text{ undoes what } g \text{ does.}}$$

If g undoes what f does and f undoes what g does, then we say that f and g are inverse functions of each other.

In mathematical notations,

If $\boxed{g(f(x)) = x}$ and $\boxed{f(g(x)) = x}$, then we say that f and g are inverse functions of each other

E.g. (a) $f(x) = 2x + 5$; $g(x) = \frac{1}{2}(x-5)$

Determine whether f and g are inverses of each other.

$$g(f(x)) = \frac{1}{2}\left((2x+5) - 5\right) = \frac{1}{2}\cdot(2x) = x$$

$$f(g(x)) = 2\left[\frac{1}{2}(x-5)\right] + 5$$

$$= (x-5) + 5 = x$$

Yes, f and g are inverses of each other.

(b) $f(x) = \dfrac{x}{9+x}$; $g(x) = \dfrac{9x}{1-x}$.

Determine whether f and g are inverses of each other.

$$g(f(x)) = g\left(\frac{x}{9+x}\right) = \frac{9\cdot\frac{x}{9+x}}{\frac{1}{1}\frac{(9+x)}{(9+x)} - \frac{x}{9+x}}$$

$$= \frac{\frac{9x}{9+x}}{\frac{9+x-x}{9+x}} = \frac{\frac{9x}{9+x}}{\frac{9}{9+x}} = \frac{\cancel{9}x}{\cancel{9+x}}\cdot\frac{\cancel{9+x}}{\cancel{9}} = x$$

$$f(g(x)) = f\left(\frac{9x}{1-x}\right) = \frac{\frac{9x}{1-x}}{\frac{(1-x)\,9}{(1-x)\cdot 1} + \frac{9x}{1-x}} = \frac{\frac{9x}{1-x}}{\frac{9(1-x)+9x}{1-x}}$$

$$= \frac{\frac{9x}{1-x}}{\frac{9-\cancel{9x}+\cancel{9x}}{1-x}} = \frac{\frac{9x}{1-x}}{\frac{9}{1-x}} = \frac{\cancel{9}x}{\cancel{1-x}} \cdot \frac{\cancel{1-x}}{\cancel{9}} = x$$

$\longrightarrow$ Yes, they are inverses of each other.

Important Notation: If g is the inverse of f, then we write $g = f^{-1}$. (Note: $f^{-1}(x)$ Does NOT mean $\frac{1}{f(x)}$)

Eg. (a) $g(x) = \frac{1}{2}(x-5)$; $f(x) = 2x+5$. g and f are inverses

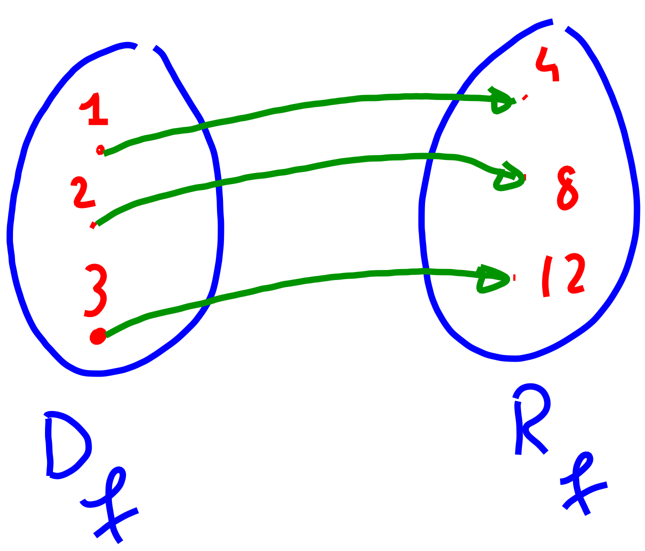
We write $f^{-1}(x) = \frac{1}{2}(x-5)$

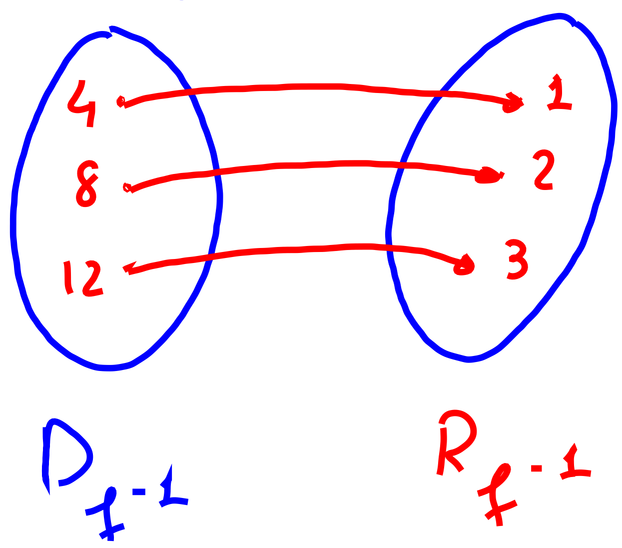
* Find the formula for the inverse functions of a one-to-one function.

Note: Only functions that are one-to-one on its domain of consideration have inverses. Why?

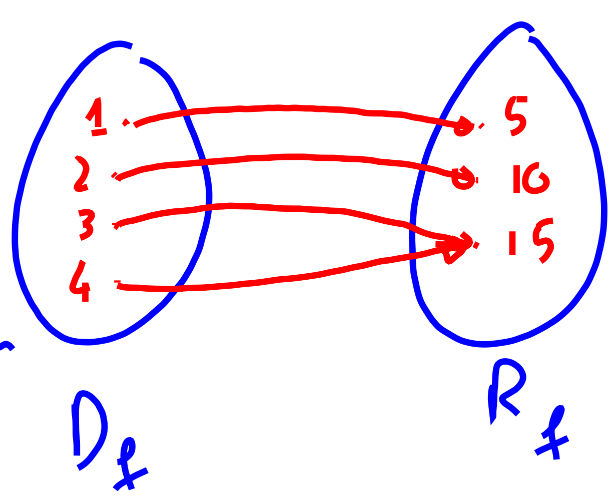
f

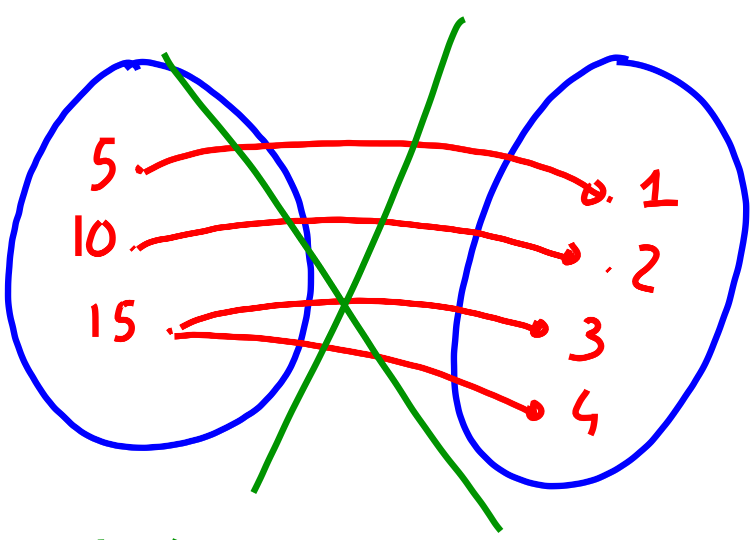
1-1 function





f: NOT a 1-1 function





NOT a function.

Note: $R_f = D_{f^{-1}}$; $D_f = R_{f^{-1}}$.

Find formulas for inverse function.

E.g. $f(x) = \frac{1}{3}(x-5)$

Q: Find a formula for $f^{-1}(x)$.

Process for finding f^{-1}

Step 1: Replace $f(x)$ by y

Step 2: Solve for x in terms of y. (get x by itself)

Step 3: Exchange x and y

Step 4: Replace y by $f^{-1}(x)$

1. $y = \frac{1}{3}(x-5)$
2. $3y = x - 5$

   $3y + 5 = x$

   $x = 3y + 5$
3. $y = 3x + 5$
4. $\boxed{f^{-1}(x) = 3x + 5}$

E.g. $f(x) = \dfrac{2}{x-3} + 4.$

Find the formula for $f^{-1}(x)$

Step 1: $y = \dfrac{2}{x-3} + 4$ (Replace $f(x)$ by y)

Step 2: $y - 4 = \dfrac{2}{x-3}$

$\dfrac{(x-3)\cancel{(y-4)}}{\cancel{y-4}} = \dfrac{2}{y-4}$

$x - 3 = \dfrac{2}{y-4}$

$\boxed{x = \dfrac{2}{y-4} + 3}$

Get x by itself

Step 3: $y = \dfrac{2}{x-4} + 3$ (Exchange x and y)

Step 4: $\boxed{f^{-1}(x) = \dfrac{2}{x-4} + 3}$

E.g $f(x) = 2 + \sqrt{x-4}$.

Find formula for $f^{-1}(x)$.

Step 1: $y = 2 + \sqrt{x-4}$

Step 2: $(y-2)^2 = (\sqrt{x-4})^2$

$(y-2)^2 = x - 4$

$(y-2)^2 + 4 = x$

$x = (y-2)^2 + 4$

Get x by itself.

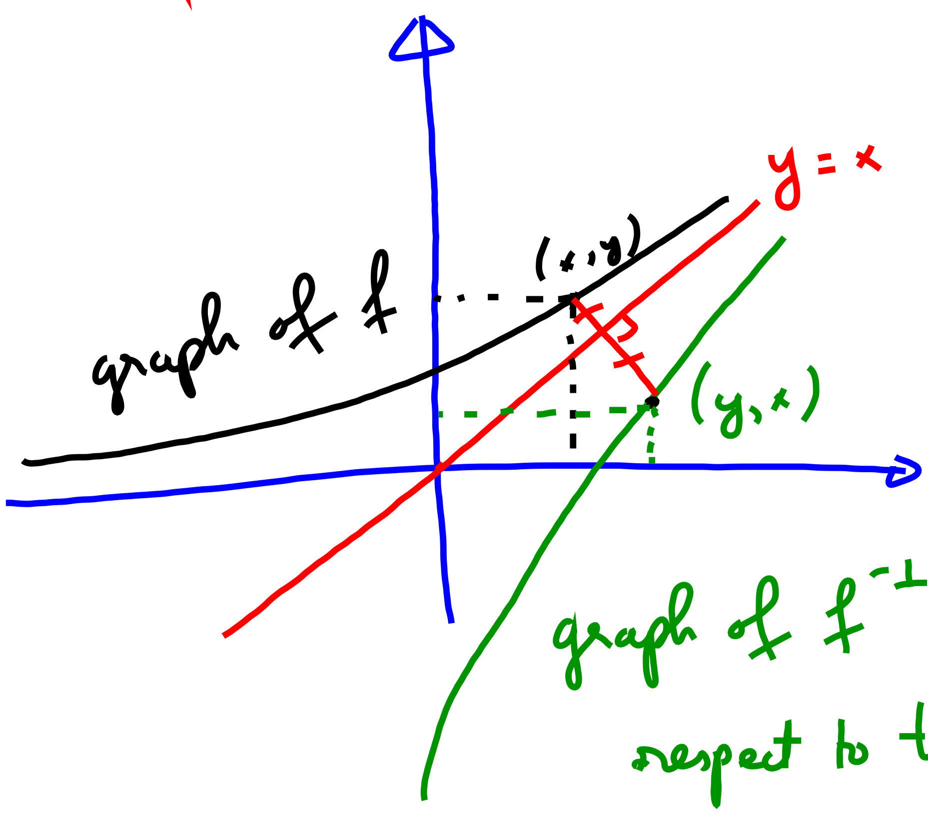
Step 3: $y = (x-2)^2 + 4$

Step 4: $\boxed{f^{-1}(x) = (x-2)^2 + 4}$

Evaluate f^{-1}. Find domain & range of f^{-1}

Did HW 10, 11, 12, 13

Relationship btwn the graph of f and graph f^{-1}



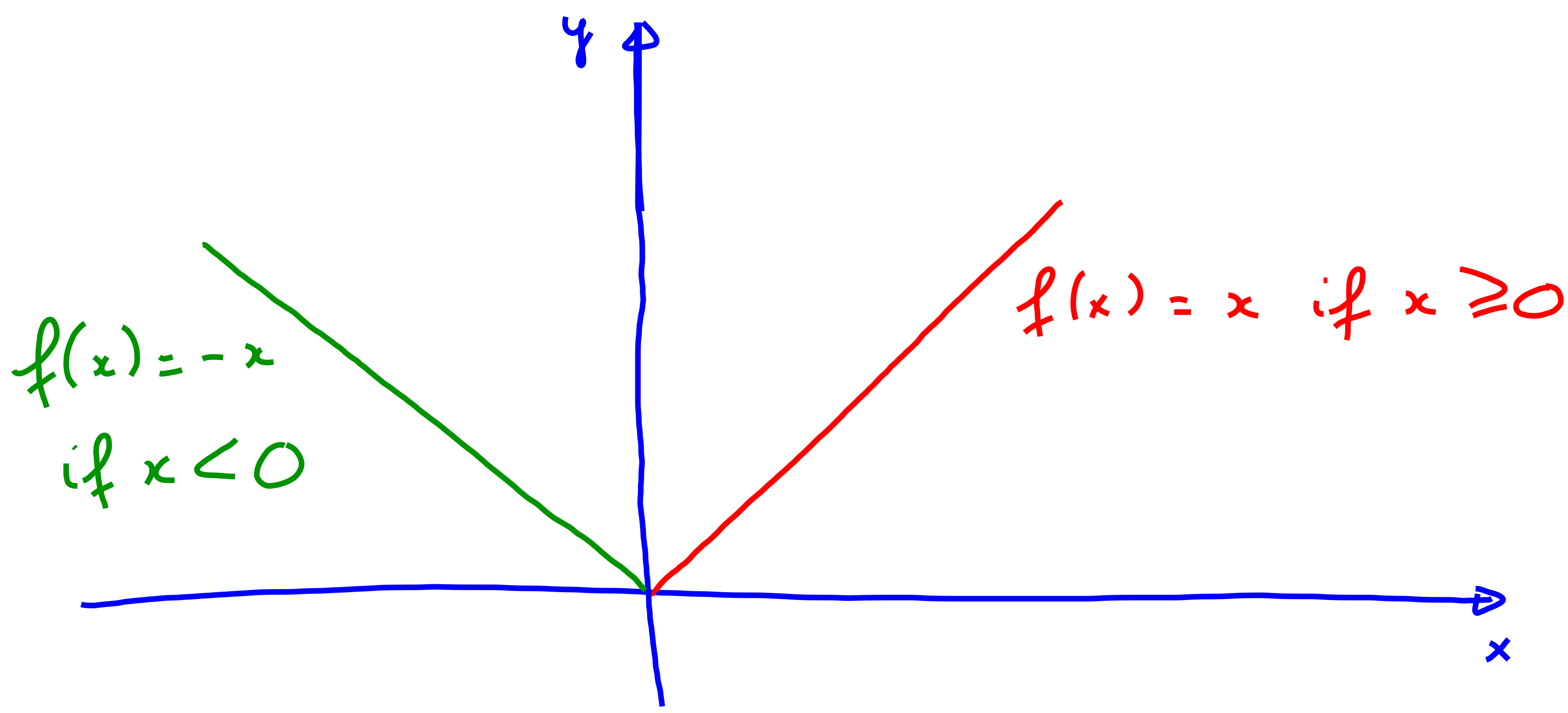
(x,y) is on the graph of f $\rightarrow$ (y,x) is on the graph of f^{-1}

graph of f^{-1} is symmetric with respect to the line $y=x$ with graph f

Section 3.6

Graph of the absolute value function

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Graph the function $g(x) = |x+2| + 3$

Process: $g(x) = f(x+2) + 3$ where $f(x) = |x|$.

So to graph g, we move graph of f up by 3, left by 2.

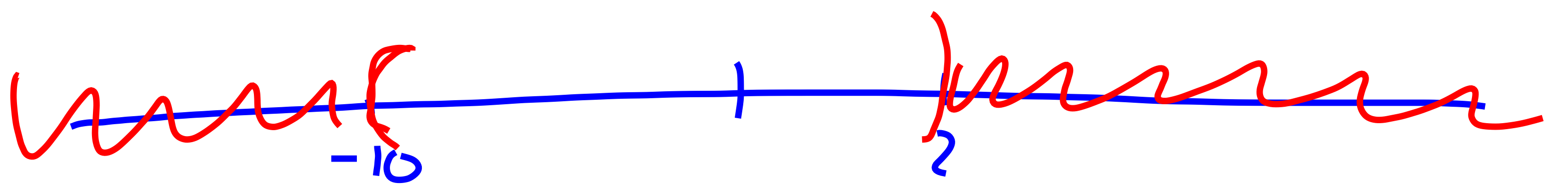
Solve absolute value inequalities

E.g. $|3x + 12| < 18$

This is equivalent to: $-18 < 3x + 12 < 18$

$3x + 12 < 18 \Rightarrow 3x < 6 \Rightarrow x < 2$

$3x + 12 > -18 \Rightarrow 3x > -30 \Rightarrow x > -10$



Solution set: $(-10, 2)$