

4.1. Linear Functions

Goals: ① linear functions and their graphs

② Slope of a line passing through 2 points.

③ Point-Slope equations and Slope-intercept equation of linear functions

④ Parallel and Perpendicular lines

⑤ Webassign HW (if time allows).

A linear function is a function of the form

$$f(x) = mx + \boxed{b}; \quad m, b \text{ are constants.}$$

E.g. $f(x) = 2x - 1$

$$\boxed{g(x) = 4x - 6}$$

$\rightarrow$ y-coordinate of y-intercept.

The graph of a linear function is a straight line.

Graph $g(x) = 4x - 6$.

$$x\text{-intercept: } 4x - 6 = 0$$

$$x = \frac{3}{2}$$

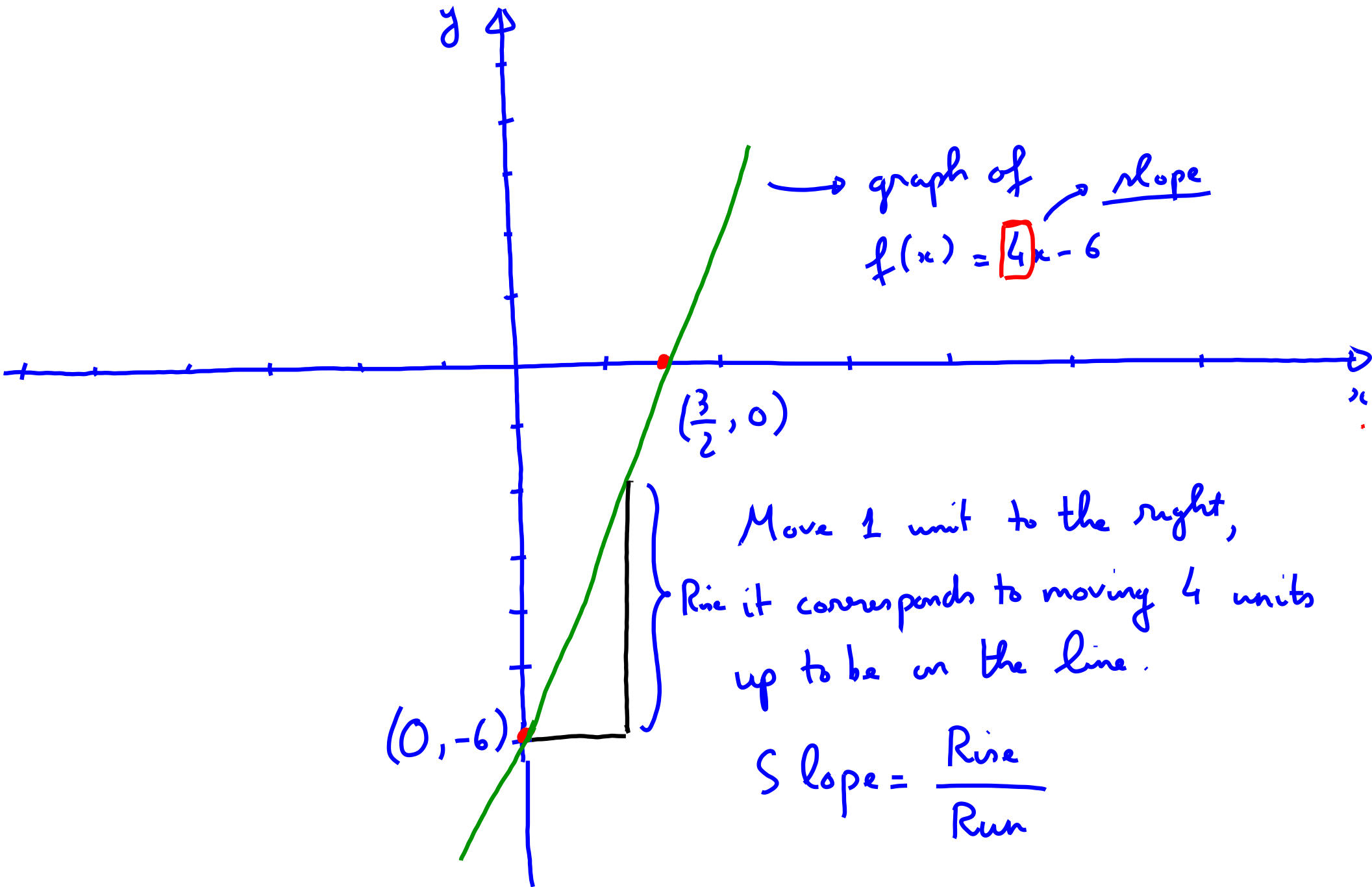
x	y
$\frac{3}{2}$	0
0	-6

$\rightarrow$ Point $\boxed{(\frac{3}{2}, 0)}$

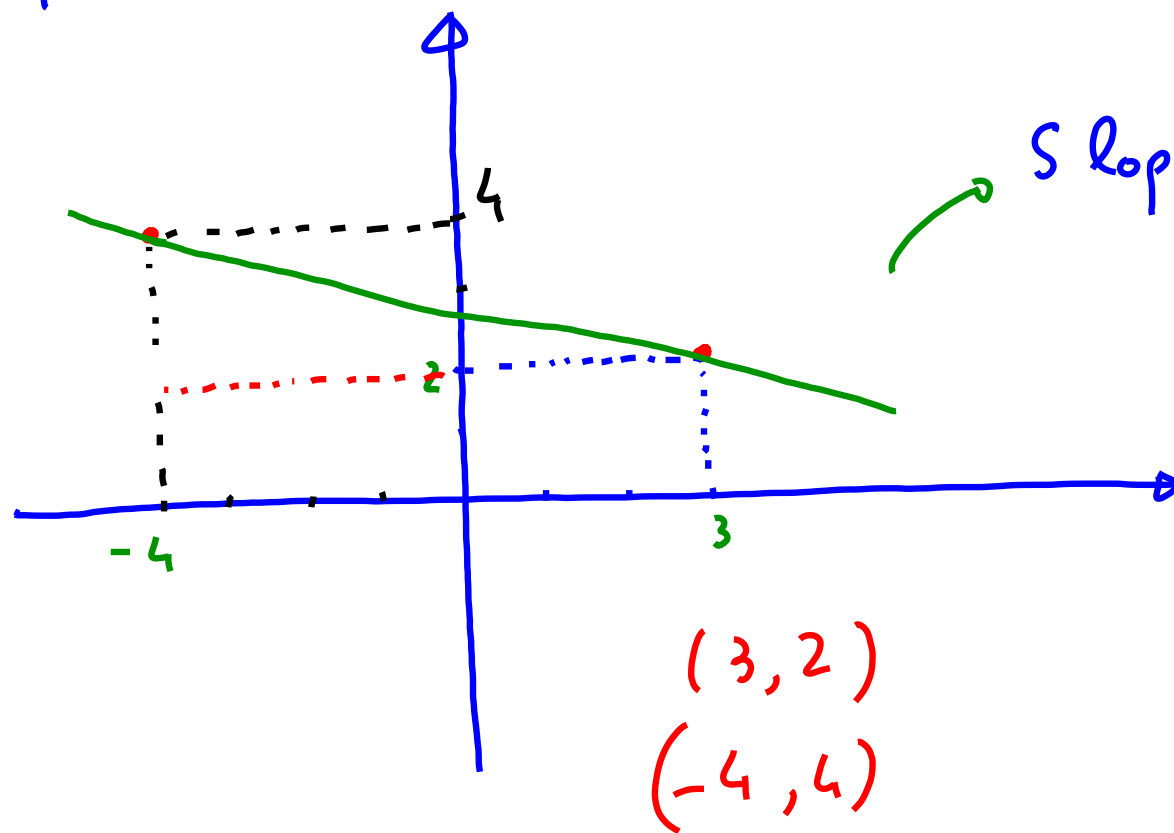
$$y\text{-intercept: Set } x = 0 \text{ in formula.}$$

$$y = 4 \cdot 0 - 6 = -6$$

$\rightarrow$ Point $\boxed{(0, -6)}$



Slope of a line that passes through 2 points:



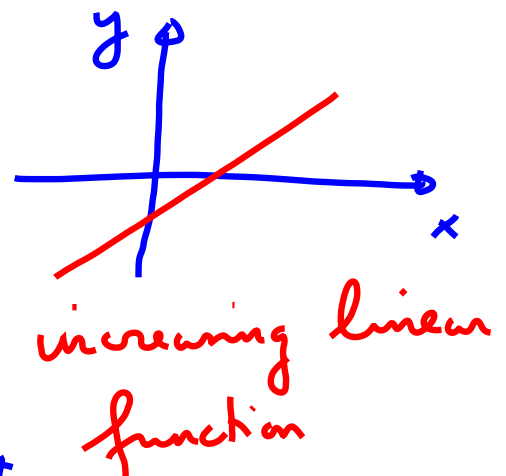
$$\begin{aligned} \text{Slope} &= \frac{4 - 2}{-4 - 3} \\ &= \frac{2}{-7} = -\frac{2}{7} \end{aligned}$$

In general, the slope of the line passing through 2 points (x_1, y_1) and (x_2, y_2) is given by the formula:

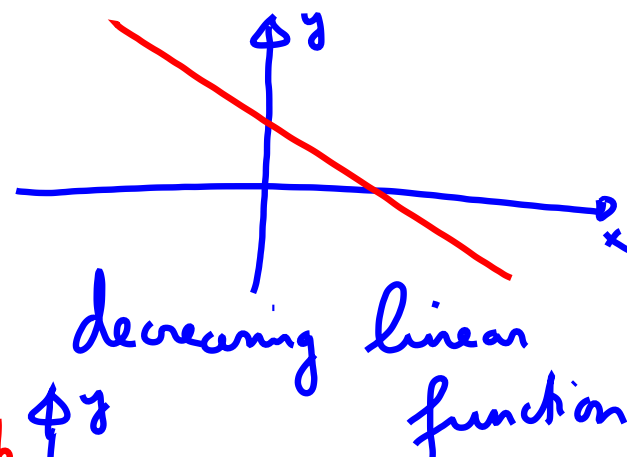
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{Rise}}{\text{Run}}$$

$f(x) = mx + b$, m is the slope:

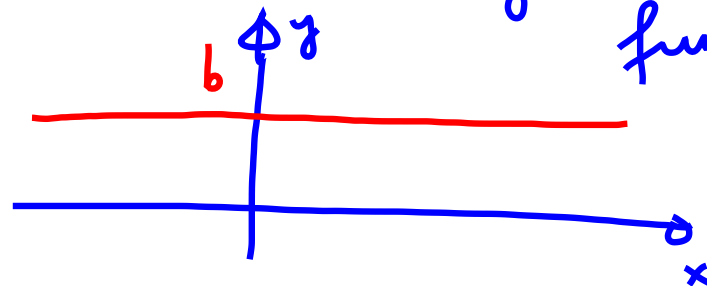
$m > 0$: the line rises from left to right



$m < 0$: the line falls from left to right.



$m = 0$: function becomes
 $f(x) = b$



Point Slope equation and slope intercept equation of a linear function.

2 points $(3, 2)$; $(-4, 4)$.

We calculated slope: $m = -\frac{2}{7}$.

Find the equation for the linear function corresponding to this line.

$$y = mx + b ; y = -\frac{2}{7}x + b.$$

$(3, 2)$ belongs to the line.

$$2 = -\frac{2}{7} \cdot 3 + b$$

$$y = -\frac{2}{7}x + \frac{20}{7}$$

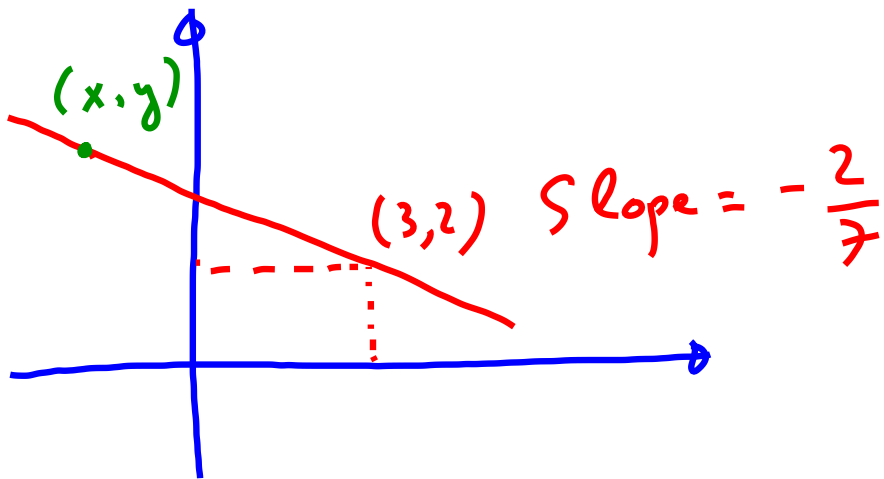
Slope - intercept equation.

$$2 = -\frac{6}{7} + b$$

$$2 + \frac{6}{7} = b$$

$$b = \frac{20}{7}.$$

Point-Slope Equation:



$$\frac{y - 2}{x - 3} = -\frac{2}{7}$$

$$y - 2 = -\frac{2}{7}(x - 3)$$

Point-slope equation.

$$y - 2 = -\frac{2}{7}x + \frac{6}{7}$$

$$y = -\frac{2}{7}x + \frac{6}{7} + 2$$

$$y = -\frac{2}{7}x + \frac{20}{7} \leftarrow \text{slope intercept}$$

General formula for point-slope equation of the line
passing through (x_1, y_1) , (x_2, y_2)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

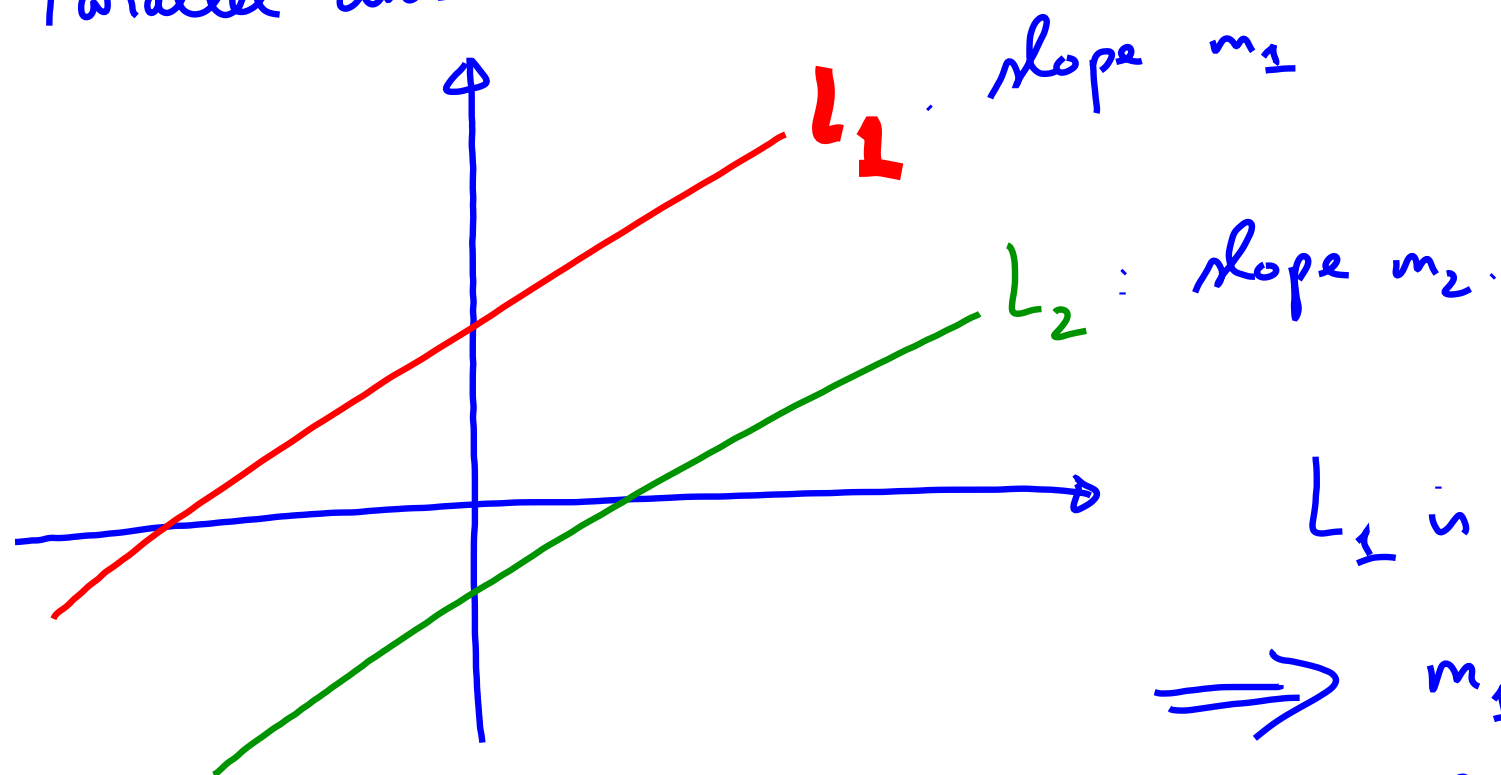
Point-Slope equation: $y - y_1 = m(x - x_1)$

Simplify this equation, i.e., get y by itself

you will get the slope intercept equation:

$$y = mx + b$$

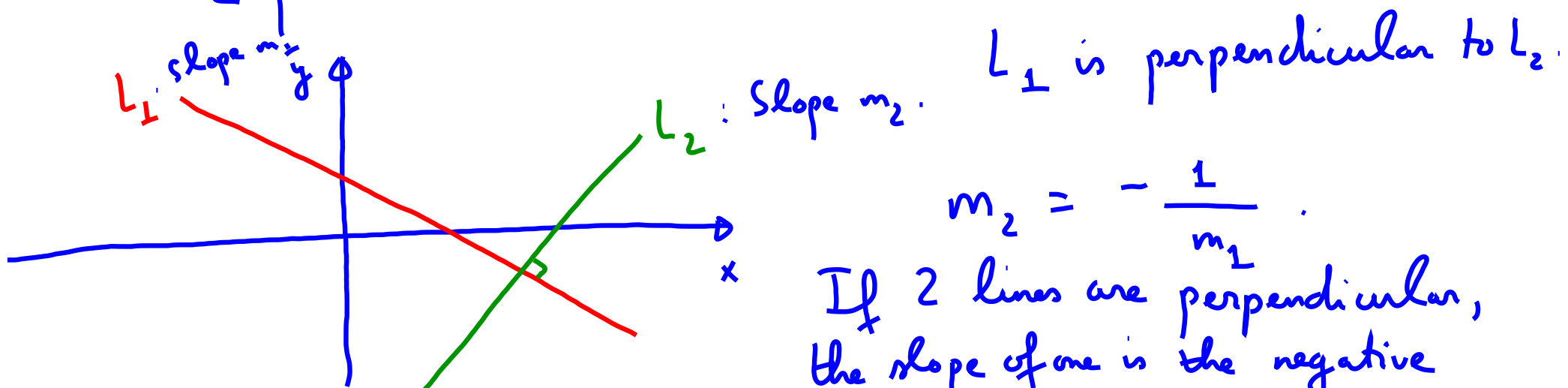
Parallel lines



L_1 is parallel to L_2 .

$$\Rightarrow m_1 = m_2$$

2 parallel lines have the same slope.



L_1 is perpendicular to L_2 .

$$m_2 = -\frac{1}{m_1}$$

If 2 lines are perpendicular, the slope of one is the negative reciprocal of the slope of the other.

#20.

$g(x)$ is parallel to $f(x)$. Hence slope of $g(x)$ is -6 .

Pt-Slope formula:

$$y - (-17) = -6(x - 3)$$

$$y + 17 = -6x + 18$$

$$y = -6x + 1$$

$$\boxed{g(x) = -6x + 1}$$