

5.1. Quadratic Functions

Goals: (1) Vertex of a parabola.

(2) Graph Quadratic functions

(3) Find max/min of quadratic functions

(4) Standard Form of a quadratic function

(5) Find domain/range of a quadratic function.

Recall: linear function $f(x) = mx + b$, ← general formula for a linear function

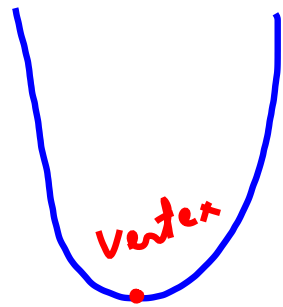
A quadratic function is a function of the form:

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

$$\left. \begin{array}{l} \text{E.g. } f(x) = 4x^2 + 2x + 2 \\ g(x) = -\frac{1}{3}x^2 - \frac{5}{7}x + \frac{12}{17} \end{array} \right\} \text{Examples of quadratic function}$$

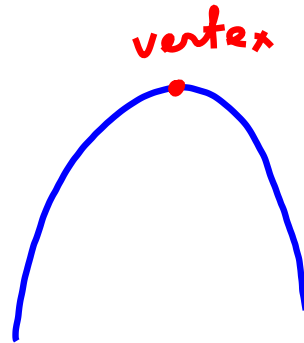
The graph of a quadratic function is a parabola.

There are only 2 kinds of parabolas in the world



upward

$$a > 0$$



downward

$$a < 0$$

→ Formula for the x and y coordinates of the vertex

The x -coordinate of the vertex:

$$h = -\frac{b}{2a}$$

The y -coordinate of the vertex:

$$k = f\left(-\frac{b}{2a}\right)$$

The coordinates of the vertex:

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

E.g. $f(x) = x^2 + 2x + 1$. $a = 1$; $b = 2$

$$x\text{-vertex} : -\frac{b}{2a} = \frac{-2}{2 \cdot 1} = -1$$

$$y\text{-vertex} : f(-1) = (-1)^2 + 2 \cdot (-1) + 1 = 0$$

Why knowing the vertex is important?

It helps you understand everything about the quadratic function.

Graph Quadratic functions

Process: ① Find the vertex : $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

② Find the axis of symmetry : $x = -\frac{b}{2a}$.

③ Find points on both sides of the axis of symmetry and connect them to make the parabola.

#7. Web Assign HW.

Standard Form of a quadratic function

$$f(x) = ax^2 + bx + c, a \neq 0 \leftarrow \text{general form.}$$

The standard form of this quadratic function is the form

$$f(x) = a(x - h)^2 + k$$

where (h, k) is the vertex

E.g $f(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + 3 \leftarrow \text{general form.}$

Q: Write this function in standard form.

$$\underline{\text{x-vertex}} = -\frac{\frac{3}{2}}{2 \cdot \left(-\frac{1}{2}\right)} = -\frac{\frac{3}{2}}{-1} = \frac{3}{2}$$

$$\begin{aligned}
 y\text{-vertex: } f\left(\frac{3}{2}\right) &= -\frac{1}{2} \cdot \left(\frac{3}{2}\right)^2 + \frac{3}{2} \cdot \frac{3}{2} + 3 \\
 &= -\frac{1}{2} \cdot \frac{9}{4} + \frac{9}{4} + 3 \\
 &= -\frac{9}{8} + \frac{9}{4} + \frac{3}{1} \\
 &= \frac{-9 + 18 + 24}{8}
 \end{aligned}$$

$$\text{Vertex: } \left(\frac{3}{2}, \frac{33}{8}\right)$$

Standard form:

$$\begin{aligned}
 f(x) &= a(x-h)^2 + k \\
 &= \boxed{-\frac{1}{2}\left(x - \frac{3}{2}\right)^2 + \frac{33}{8}}
 \end{aligned}$$

Find max/min of a quadratic function.

Did # 15, 16.

Find Domain / Range of a quadratic function.

Did # 17, 18