5.2. Power Functions and Polynomial Functions

Understand the behavior of the graphs of these functions.

Linear function: $f(x) = mx + b$. Graph: straight line.

Quadratic function: $f(x) = ax^2 + bx + c$; $a \neq 0$.

<u>Graph</u>: $\cup$ ($a > 0$) $\quad \cap$ ($a < 0$)

<u>Vertex</u>: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Power Functions

A power function is a function of the form

$$\boxed{f(x) = k \cdot x^p}$$: k, p constants

E.g.

$f(x) = x^2$ / $k = 1$; $p = 2$

$g(x) = -3x^5$ / $k = -3$; $p = 5$

$h(x) = \dfrac{1}{x^2} = x^{-2}$ $k = 1$; $p = -2$

$k(x) = \dfrac{10}{-x^{17}}$

$= -10x^{-17}$

$k = -10$; $p = -17$

$l(x) = -3\sqrt{x} = -3 \cdot x^{1/2}$; $k = -3$; $p = \frac{1}{2}$

$m(x) = \sqrt[3]{x^5} = x^{5/3}$; $k = 1$; $p = \frac{5}{3}$

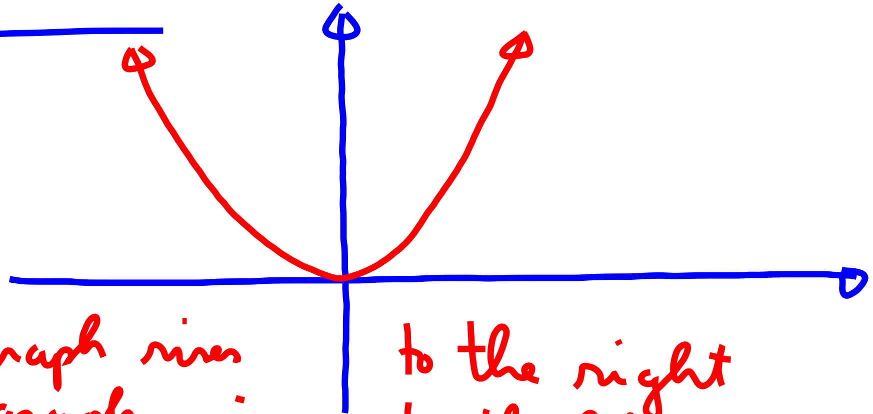
$n(x) = 2^x \rightarrow$ Not a power function

$p(x) = \dfrac{3x+1}{2x-5} \rightarrow$ Not a power function

$q(x) = 2x^3 + 5x + 6 \rightarrow$ Not a power function

End behaviors of power functions

$f(x) = x^2$



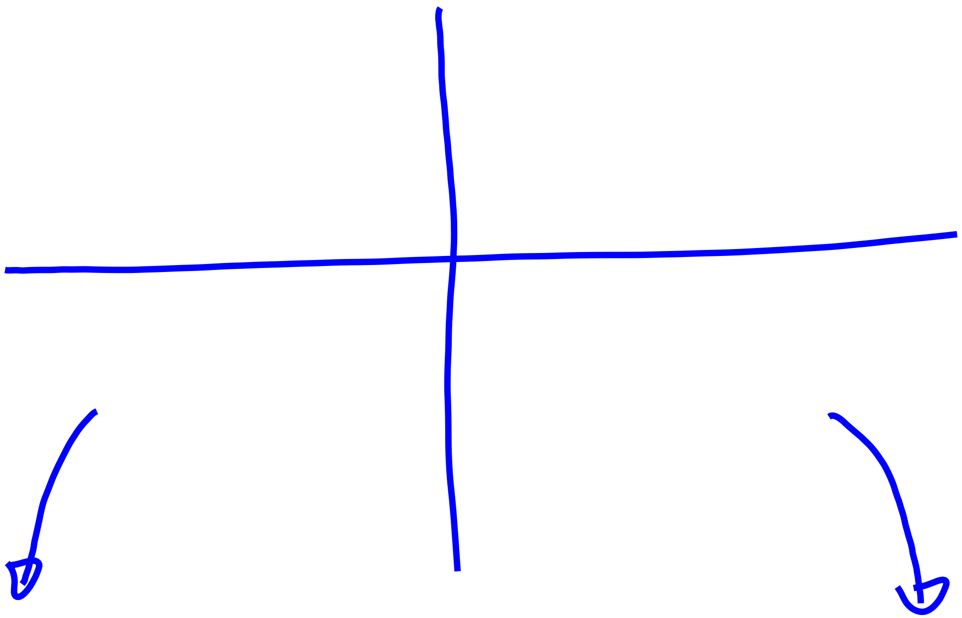
$x \rightarrow \infty$, $f(x) \rightarrow \infty$: the graph rises to the right

$x \rightarrow -\infty$, $f(x) \rightarrow \infty$: the graph rises to the left

$g(x) = \boxed{-120} \cdot x^{\boxed{50}}$ $-120x^{51}$

Determine the end behavior of this function.

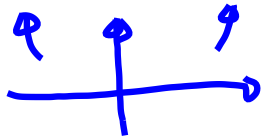
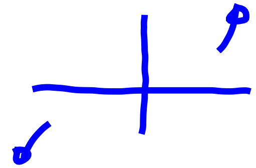
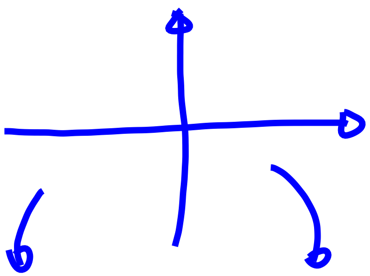
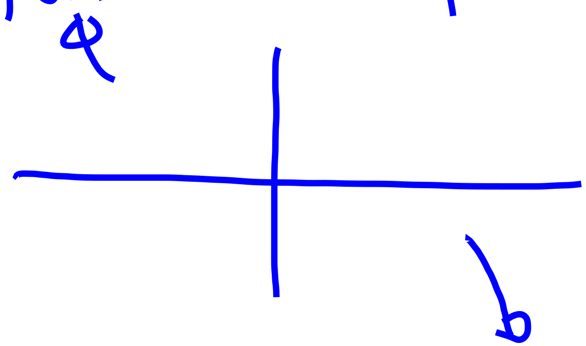
Falls to the left,



As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to -\infty$

$k \cdot x^p$

	p even	p odd
$k > 0$	Rises to the right Rises to the left 	Rises to the right Falls to the left 
$k < 0$	Falls to the right Falls to the left 	Falls to the right Rises to the left 

End Behavior of Polynomial Functions.

Some important terminology:

Degree: is the highest power in the polynomial.

leading term: is the term in the polynomial with the highest power.

leading coefficient: is the coefficient in the leading term.

Fact: (Important)

The end behavior of a polynomial matches the end behavior of its leading term.

Fact:

* If a polynomial has degree n, it has at most n x-intercepts (zero) (Root)

* If a polynomial has degree n, it has at most $n-1$ turning points.