

5.3. Graphs of Polynomial Functions

- Goals:
- ① Reminder: End behavior of polynomials
 - ② Zeros and Multiplicities of Zeros.
 - ③ Sketch graphs of polynomial functions.
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End Behavior

	ax^n	
	n even	n odd
$a > 0$		
$a < 0$		

End behavior of a polynomial function matches the end behavior of its leading term

E.g. $f(x) = 2x^4 + 3x^3 - x^2 + x - 10$

End behavior?

leading term: $2x^4$ $\longrightarrow$ end behavior

E.g. $g(x) = (2x - 1)^3 (x + 2)^2 (-x^2 + 10)$

End behavior?

leading term: $(2x)^3 \cdot x^2 \cdot (-x^2) = 8x^3 \cdot x^2 \cdot (-x^2)$
 $= -8x^7$

End behavior:

Zeros and Multiplicities

Zero of a polynomial
(Roots)

E.g. $f(x) = (x+2)^1(x-3)^1$

Find the zeros of f ?

To find the zeros of $f(x)$, we set $f(x) = 0$

$$(x+2)(x-3) = 0$$

$$x = -2 \text{ or } x = 3$$

The zeros of the function f are:

$$x = -2$$

→ a zero of multiplicity 1

$$x = 3$$

→ a zero of multiplicity 1

Ex $g(x) = (x-3)(x+2)^3(x+1)^2$

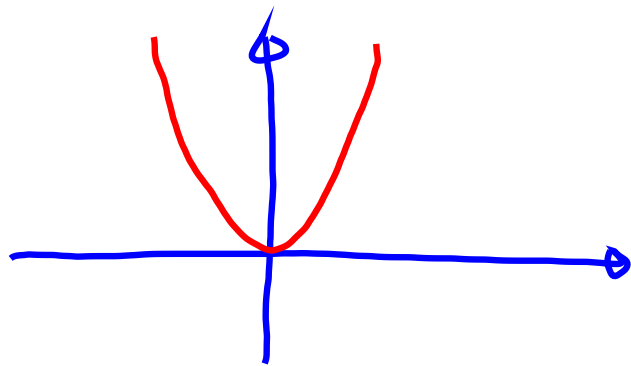
Find the zeros of g and the multiplicity of each zero.

Zeros:

$x = 3$	$\longrightarrow$	multiplicity = 1
$x = -2$	$\longrightarrow$	multiplicity = 3
$x = -1$	$\longrightarrow$	multiplicity = 2

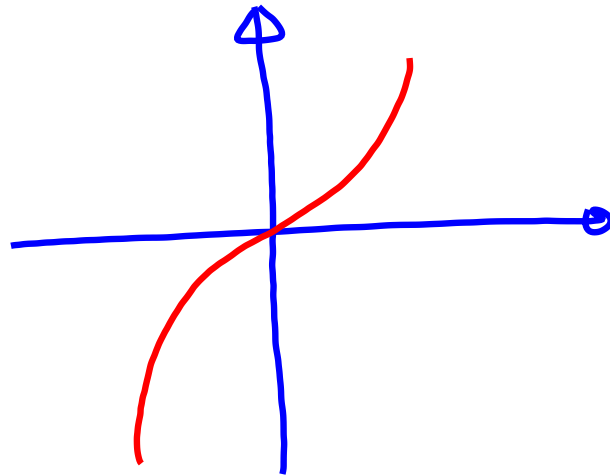
$$f(x) = x^2$$

Zero: $x = 0$; mult. = 2



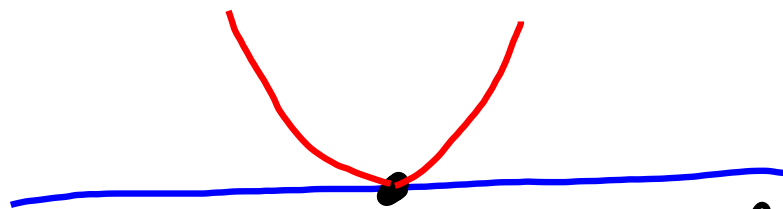
$$f(x) = x^3$$

Zero: $x = 0$; mult. = 3



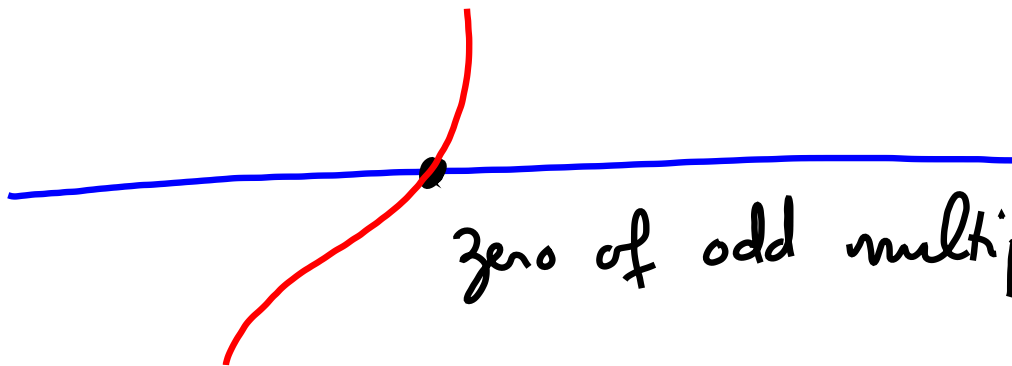
Why the multiplicity of a zero matters?

① If a zero has **EVEN** multiplicity, when the graph reaches that zero, it will touch the zero and bounce back.



zero of even multiplicity

② If a zero has **ODD** multiplicity, when the graph reaches that zero, it will cross that zero.



zero of odd multiplicity.

Put everything together to graph polynomial functions

Guideline on graphing polynomial functions.

① Find zeros (x -parts of x -intercepts) (Set $f(x) = 0$)

Find y -intercept (Set $x = 0$)

② Determine end behavior. (Find leading term)

③ Use the multiplicity of zeros to determine the behavior of the graph at zeros.

④ Put everything together to sketch the graph.

E.g. Sketch a graph of the polynomial function:

$$f(x) = -2(x+3)^2(x-2)$$

① Find x -intercepts, y -intercept.

x -intercepts: $-2(x+3)^2(x-2) = 0$

$$x = -3 ; x = 2$$

x -intercepts: $(-3, 0), (2, 0)$

y -intercept: Set $x = 0$: $-2 \cdot (3)^2 \cdot (-2) = 36$

y -intercept: $(0, 36)$

② End behavior

leading term: $-2x^3$. End behavior.

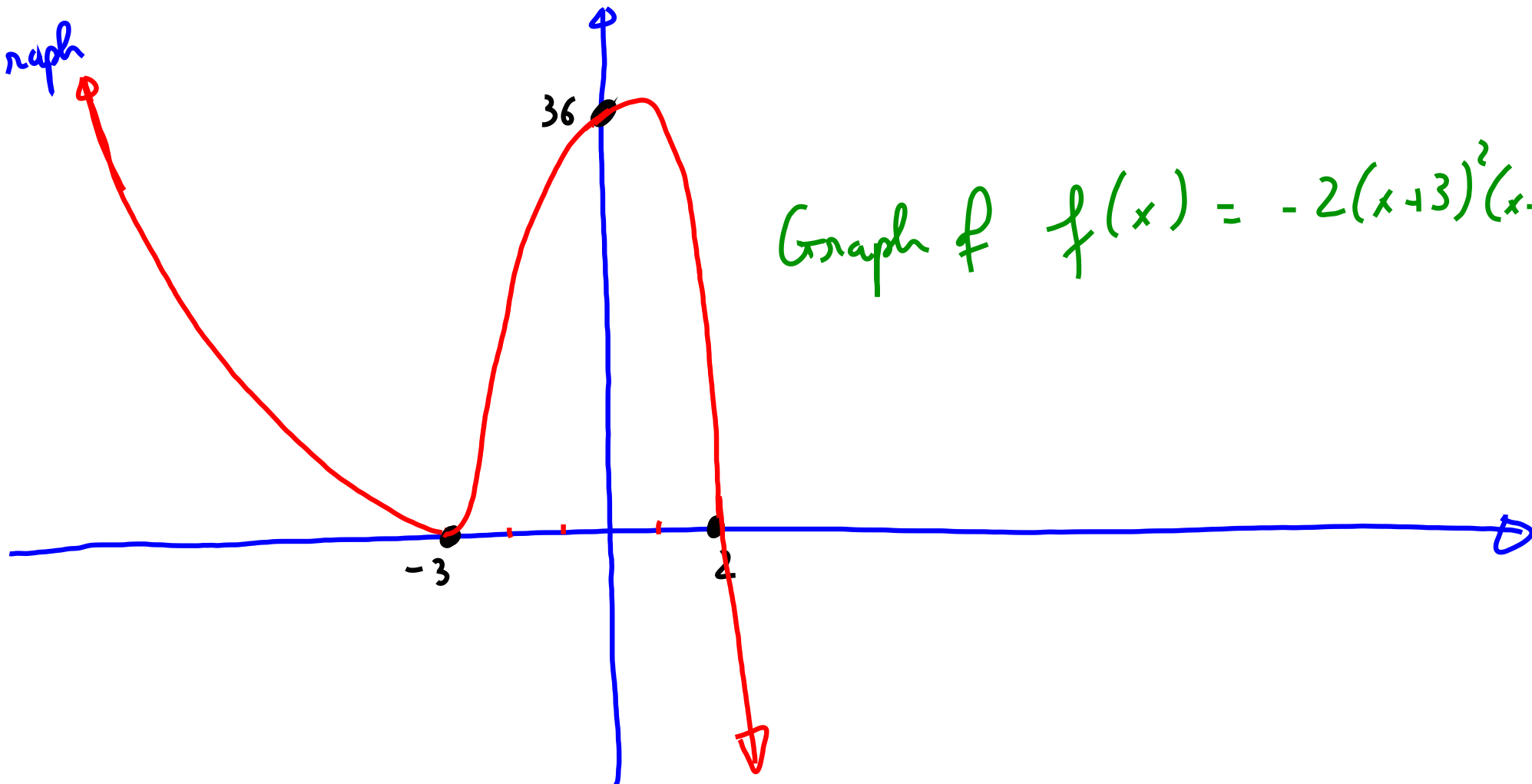


③ Multiplicity

$x = -3$. Multiplicity 2

$x = 2$. Multiplicity 1

④ Graph



Graph of $f(x) = -2(x+3)^2(x-2)$

Web assign: Did # 6 ; # 7 ; # 19