5.4. Dividing Polynomials

Goals:
1. Long Division
2. Synthetic Division

E.g. Example to illustrate the process of long division.

$$\underbrace{\left(4x^3 + 12x^2 - x - 8\right)}_{\text{Dividend}} \div \underbrace{\left(2x + 5\right)}_{\text{Divisor}}$$

$$\frac{4x^3 + 12x^2 - x - 8}{2x + 5}$$

$$\begin{array}{r l}
 & \boxed{2x^2 + x - 3} \longrightarrow \text{Quotient} \\
\boxed{2x} + 5 \,\big) & \boxed{4x^3} + 12x^2 - x - 8 \\
 & -(4x^3 + 10x^2) \\
\hline
 & \boxed{2x^2} - x - 8 \\
 & -(2x^2 + 5x) \\
\hline
 & \boxed{-6x} - 8 \\
 & -(-6x - 15) \\
\hline
 & \boxed{7} \longrightarrow \text{Remainder.}
\end{array}$$

$$\frac{4x^3}{2x} = 2x^2$$

$$\frac{2x^2}{2x} = x$$

$$\frac{-6x}{2x} = -3$$

When we divide $4x^3 + 12x^2 - x - 8$ by $2x + 5$, the quotient is $\boxed{2x^2 + x - 3}$ and the remainder is $\boxed{7}$

$$\frac{4x^3 + 12x^2 - x - 8}{2x+5} = 2x^2 + x - 3 + \frac{7}{2x+5}$$

$$\underbrace{4x^3 + 12x^2 - x - 8}_{\text{Dividend}} = \underbrace{(2x^2 + x - 3)}_{\text{Quotient}}\underbrace{(2x+5)}_{\text{Divisor}} + \underbrace{7}_{\text{Remainder}}$$

$6x^6 + 3x^3 + x - 18 \longleftarrow$ Dividend

$3x^2 + 2x - 1 \longleftarrow$ Divisor

$$\frac{6x^6 + 3x^3 + x - 18}{3x^2 + 2x - 1}$$

$$2x^4 - \frac{4}{3}x^3 + \frac{14}{9}x^2$$

$$3x^2 + 2x - 1 \,\big)\, \boxed{6x^6} + 3x^3 + x - 18$$

$$-(6x^6 + 4x^5 - 2x^4)$$

$$\boxed{-4x^5} + 2x^4 + 3x^3 + x - 18$$

$$-\left(-4x^5 - \frac{8}{3}x^4 + \frac{4}{3}x^3\right)$$

$$\boxed{\frac{14}{3}x^4} + \frac{5}{3}x^3 + x - 18$$

$$\frac{6x^6}{3x^2} = 2x^4$$

$$\frac{-4x^5}{3x^2} = -\frac{4}{3}x^3$$

$$\frac{\frac{14}{3}x^4}{3x^2} = \frac{14}{9}x^2$$

...... HW.

Synthetic Division

When the divisor has the form $x - a$ or $x + a$ where a is constant, we can apply synthetic division.

E.g. Situation when the divisor has the form $x - a$.

$$(-9x^4 + 10x^3 + 7x^2 - 6) \div (x - 1)$$

using synthetic division.

$$\begin{array}{r|rrrrr}
1 & -9 & 10 & 7 & 0 & -6 \\
  &   & -9 & 1 & 8 & 8 \\
\hline
  & -9 & 1 & 8 & 8 & \boxed{2}
\end{array} \longrightarrow \text{Remainder}$$

Quotient: $\boxed{-9x^3 + x^2 + 8x + 8}$

$$\frac{-9x^4+10x^3+7x^2-6}{x-1} = \left(-9x^3+x^2+8x+8\right) + \frac{2}{x-1}$$

E.g. Situation where the divisor has the form $x+a$.

$$\left(3x^4+18x^2-3x+40\right) \div \left(x+2\right)$$

$$\begin{array}{r|rrrrr}
-2 & 3 & 0 & 18 & -3 & 40 \\
 & & -6 & 12 & -60 & 126 \\
\hline
 & 3 & -6 & 30 & -63 & 166
\end{array}$$

Remainder: 166

Quotient: $3x^3-6x^2+30x-63$

$$\frac{3x^4+18x^2-3x+40}{x+2} = 3x^3-6x^2+30x-63+\frac{166}{x+2}$$

Web Assign.

Did 7, 8, 10