6.1. and 6.2 Exponential Functions and their graphs

Goals: (1) Define exponential Functions. Evaluate Exponential Functions.

(2) Model using Exp. Functions

(3) Graphs of Exp. Functions

Definition of an exponential function.

An exponential function is a function of the form

$$f(x) = a \cdot (b)^x$$

a is a nonzero constant. It is called the initial value of the function.

b is a nonzero constant, $b \neq 1$. It is called the base of the function.

E.g.

$$f(x) = 4 \cdot (2)^x$$

$$f(0) = 4 \quad ; \quad f(4) = 4 \cdot 2^4 = 64$$

$$f(-3) = 4 \cdot (2)^{-3} = 4 \cdot \frac{1}{2^3} = 4 \cdot \frac{1}{8} = \frac{1}{2}$$

Key: $(b)^{-x} = \frac{1}{b^x}$

$$f\left(\frac{2}{3}\right) = 4 \cdot (2)^{\frac{2}{3}} = 4 \cdot \sqrt[3]{2^2} = 4 \cdot \sqrt[3]{4}$$

$$f\left(\frac{1}{2}\right) = 4 \cdot (2)^{\frac{1}{2}} = 4 \cdot \sqrt{2}$$

Key: $(b)^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$

E.g. where an exp. function arises in practice.

$10,000 to put in an investment account

Annual return of 7%

How much money will you get after 30 years?

After 1 year. $10,000 + 10,000(0.07)$

$$= 10,000 \cdot (1 + 0.07) = 10,000 \cdot (1.07)$$

After 2 years. $10,000 \cdot (1.07)^2$

$\vdots$

After 30 years: $10,000 \cdot (1.07)^{30} = \$76,122.55$

After x years: $f(x) = 10,000 \cdot (1.07)^x$

Use exponential function to model growth / decay.

E.g. Find the formula for an exp function whose graph passes through $(0, 6)$ and $(3, 384)$

Sol: $f(x) = a \cdot (b)^x$

$y = a \cdot (b)^x$

$(0, 6)$ is on the graph $\longrightarrow$ initial value $a = 6$

$y = 6 \cdot (b)^x$

$(3, 384)$ is on the graph $\longrightarrow$ $x = 3$; $y = 384$

$384 = 6 \cdot (b)^3$; $b^3 = 64$; $b = 4$

$$\boxed{f(x) = 6 \cdot (4)^x}$$

#5. Find exp. $y = a(b)^x$ that passes through $(2, 1)$ and $(3, 8)$

$1 = a \cdot b^2$ (b/c $(2,1)$ is on the graph)

$8 = a \cdot b^3$ (—— $(3,8)$ ——————)

From the first equation: $a = \frac{1}{b^2}$

Plug this into second equation: $8 = \frac{1}{b^2} \cdot b^3 = \frac{b^3}{b^2} = b$

So, $b = 8$; $a = \frac{1}{b^2} = \frac{1}{64}$

$$\boxed{y = \frac{1}{64} \cdot (8)^x}$$

#9. Car : sold for 16,000

After 1 year : 12,000

Exp. $y = a(b)^x$

$x = 0$; $\underbrace{y = 16,000}$; $\underbrace{x = 1 \, , \, y = 12,000}$

$a = 16,000$

$12,000 = 16,000 \cdot (b)^1$

$b = \frac{3}{4}$.

$y = 16,000\left(\frac{3}{4}\right)^x$

After 8 years ; $y = 16,000 \cdot \left(\frac{3}{4}\right)^8 \approx 1602$.

Compound Interest Formula

$$A(t) = P\left(1 + \frac{R}{n}\right)^{nt}$$

P : initial amount in the account (Principal)

R : interest rate

n : compounding frequency
- annually / yearly : $n = 1$
- semi-annually : $n = 2$
- monthly : $n = 12$
- quarterly : $n = 4$

t : # years

$A(t)$: value (balance) of the account after t years.

Continuously Compound Interest Formula

$$A(t) = P \cdot e^{Rt}$$

e is constant. $e \approx 2.718282$

$$A(t) = P \cdot (2.718282)^{Rt}$$