6.3. Logarithmic Functions

Goals: (1) Define logarithmic functions

(2) Evaluate logarithmic functions

(3) Graphs of Log Functions.

E.g.

Exponential function $y = 2^x$; $2^3 = 8$

Definition of a logarithmic function

Let $b > 0$, $b \neq 1$ (b is called the base)

The logarithmic function with base b is denoted by

$\log_b x$ (read as log base b of x)

The function $\log_b x$ gives us the exponent y such that $b^y = x$

E.g. $b = 2$

$\log_2(x)$

$x = 4$: $\log_2(4) = 2$ because $2^2 = 4$

$\log_2(8) = 3$ because $2^3 = 8$

$\log_2(64) = 6$ because $2^6 = 64$

$\log_2\left(\frac{1}{2}\right) = -1$ because $2^{-1} = \frac{1}{2}$

$\log_2\left(\frac{1}{16}\right) = -4$ because $2^{-4} = \frac{1}{16}$

$\log_2(\sqrt{2}) = \frac{1}{2}$ because $2^{\frac{1}{2}} = \sqrt{2}$

$\log_2(\sqrt[3]{4}) = \frac{2}{3}$ because $\sqrt[3]{4} = \sqrt[3]{2^2} = 2^{\frac{2}{3}}$

Key: log functions give you the exponent.

Recall.

$$b^{-x} = \frac{1}{b^x} \quad ; \quad \sqrt[n]{b^m} = b^{\frac{m}{n}}.$$

Ex.

$\log_{10}(1000) = 3$

$\log_{10}(1) = 0$

$\log_{10}(10^6) = 6$

$\log_{10}(0.1) = -1$

$\log_{10}(0.000001) = -6$

$\log_{10}(0.01) = -2$

$\log_5(\sqrt{5}) = \frac{1}{5}$

$\log_5(\sqrt[3]{25}) = \frac{2}{3}.$

Common logarithm.

$\log_{10} x$ is denoted by $\log x$ (read as log of x)

E.g. $\log(100) = 2$.

Natural logarithm:

$\log_e x$ is denoted by $\ln x$

$(e \approx 2.71828\ldots)$

E.g. $\ln(1) = \log_e(1) = 0$; $\ln(\sqrt{e}) = \frac{1}{2}$

$\ln(e) = \log_e(e) = e$;

$\ln(e^2) = 2$

6.4. Graphs of Log Functions

Domain of logarithmic functions

$\log(-10) \stackrel{?}{=}$ DNE

In general $\log(x)$ DNE if $x \leq 0$

In other words, for $\log(x)$ to exist, $x > 0$

What is the domain of $f(x) = \log(x)$?

$(0, \infty)$

In general, the domain of $f(x) = \log_b(x)$ is $(0, \infty)$

or $x > 0$

E.g. $f(x) = \log_2(\boxed{2x-3})$

Find the domain of f?

To find domain: $2x - 3 > 0$

$2x > 3$

$x > \frac{3}{2}$

Domain: $\left(\frac{3}{2}, \infty\right)$

$f(x) = \log_2(2x-3)$.

Find $f(2)$? $\log_2(2 \cdot 2 - 3) = \log_2(1) = 0$

Ex. Find the domain of $f(x) = \log(\boxed{-7+x})$.

$$-7 + x > 0$$

$$x > 7$$

Domain : $(7, \infty)$

Key: To find the domain of $f(x) = \log_b(\boxed{\text{Stuff}})$,

we solve the inequality : $\text{Stuff} > 0$

E.g. $g(x) = \ln\left[\boxed{(2x-3)(x-1)}\right]$.

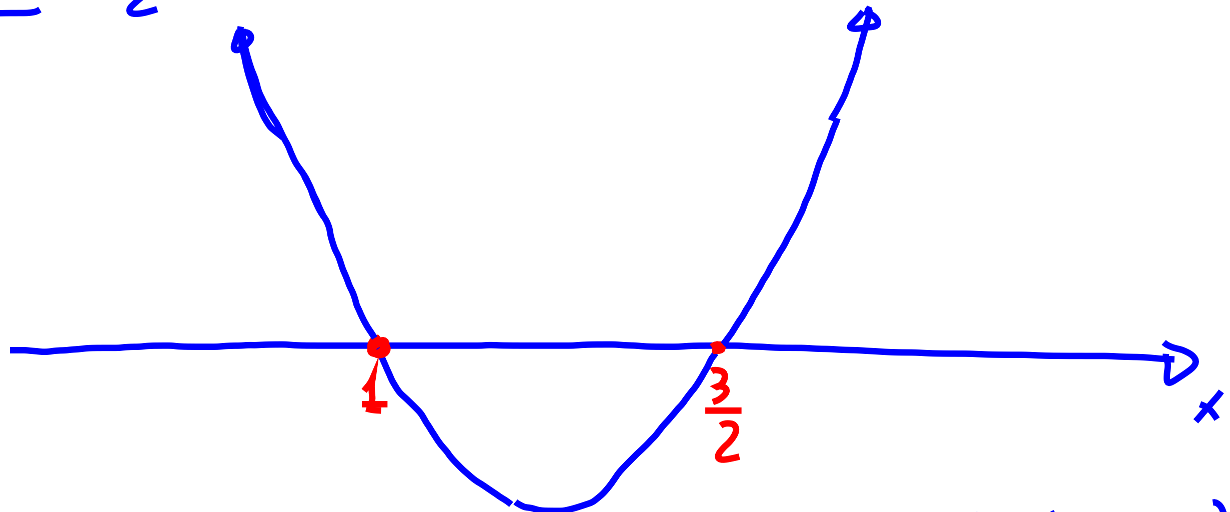
Find the domain of g?

To find the domain, we need to solve $\underbrace{(2x-3)(x-1)} > 0$

Graph $(2x-3)(x-1)$.

Leading term: $2x^2$. End Behavior $\uparrow \quad \uparrow$

<u>Zeros</u>: $\frac{3}{2}$, 1. Mult. 1



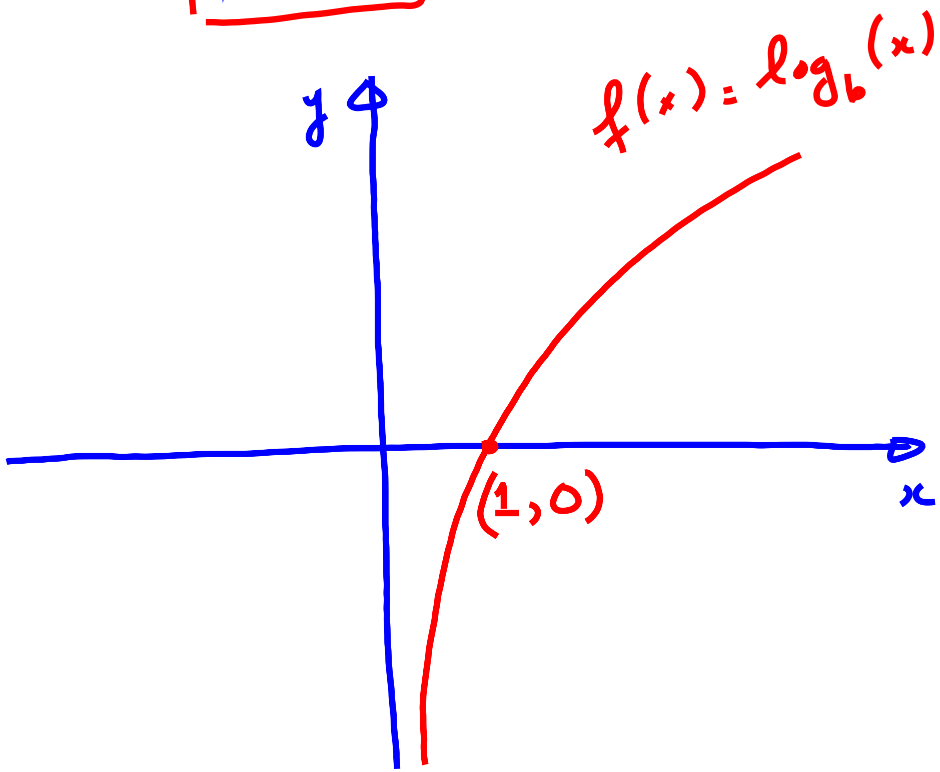
Solution to the inequality: $(2x-3)(x-1) > 0$

is $(-\infty, 1) \cup (\frac{3}{2}, \infty)$

<u>Domain</u>: $(-\infty, 1) \cup (\frac{3}{2}, \infty)$

Shapes of graphs of $f(x) = \log_b x$

$\boxed{b > 1}$



$\boxed{0 < b < 1}$