6.5. Logarithmic Properties

Goals:
1. Inverse Property
2. Product Rule
3. Power Rule
4. Quotient Rule
5. Change of base formula

Inverse Property

E.g. $\log(1000) = 3$

$\left(\log_{10}(10^{\boxed{3}}) = 3\right)$

$\log_2(64) = 6$

$\left(\log_2(2^{\boxed{6}}) = 6\right)$

$$\boxed{\log_b(b^x) = x}$$

E.g.

$\boxed{2}^{\log_{\boxed{2}}(\boxed{8})} = 2^3 = 8$

$\boxed{10}^{\log_{\boxed{10}}(\boxed{100})} = 10^2 = 100$

$$\boxed{b^{\log_b(x)} = x}$$

In general,

$$\boxed{\begin{aligned} \log_b\left(b^{\text{Stuff}}\right) &= \text{Stuff} \\ b^{\log_b(\text{Stuff})} &= \text{Stuff} \end{aligned}}$$

} Inverse Property.

E.x. Simplify

$$\ln\left(e^{2x+5}\right) = 2x+5$$

$$3^{\log_3(5x^2+9)} = 5x^2+9$$

$$3^{\log_3(x)+2} = 3^{\log_3(x)}\,3^2 = 9x$$

$$\left(b^{x+y} = b^x \cdot b^y\right)$$

2 Product Rule

Product Rule says that log of a product is equal to the sum of log of the individual factors in the product

$$\boxed{\log_b(M \cdot N) = \log_b(M) + \log_b(N)}$$

E.g. $\log_2(32) = \log_2(8 \cdot 4) = \log_2(8) + \log_2(4)$

$$\underbrace{\log_2(8 \cdot 4)}_{5} \overset{?}{=} \underbrace{\log_2(8)}_{3} + \underbrace{\log_2(4)}_{2} \quad \checkmark$$

$$5 = 3 + 2$$

E.g. $\log[\underbrace{7x}_{M} \cdot \underbrace{(x+1)}_{N}]$

Expand this using the product rule.

$= \log(7x) + \log(x+1)$ $\cancel{= \log(x) + \log(1)}$ WRONG

$= \log(7) + \log(x) + \log(x+1)$

③ Power Rule

$$\log_b\left(M^{\boxed{p}}\right) = p \cdot \log_b(M)$$

E.g.

$$\log_2\left(x^{\boxed{3}}\right) = 3 \cdot \log_2(x)$$

$$\log_2(\sqrt{x}) = \log_2\left(x^{\frac{1}{2}}\right) = \frac{1}{2}\log_2(x)$$

$$\log_3\left(x^7 y^5\right) = \log_3(x^7) + \log_3(y^5)$$

Product Rule

$$= 7 \cdot \log_3(x) + 5\log_3(y)$$

Power Rule

E.g. Expand the expression

$$\log\left(x^4 y^3 \sqrt[3]{x^2 y^5}\right)$$

Ans: $\frac{14}{3}\log(x) + \frac{14}{3}\log(y)$

(4) Quotient Rule

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

E.g. Expand

$$\log_2\left(\frac{x^4 y^3}{z^2}\right) = \log_2(x^4 y^3) - \log_2(z^2)$$

$$= 4\log_2(x) + 3\log_2(y) - 2\log_2(z)$$

So far

$$\log_b\left(b^x\right) = x$$

$$b^{\log_b(x)} = x$$

$$\log_b(M \cdot N) = \log_b(M) + \log_b(N)$$

$$\log_b\left(M^p\right) = p\log_b(M)$$

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

Combining and Expanding log expressions using these properties.

Change of base formula

$$\log_3(5) = \frac{\log_{10}(5)}{\log_{10}(3)} = \frac{\log(5)}{\log(3)} \approx 1.4649$$

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$$\boxed{\log_a b = \frac{\log_c(b)}{\log_c(d)}}$$