E.g.

$$f(x) = \begin{cases} \underline{x \sin \frac{1}{x} \quad \text{if } x \neq 0} \\ \boxed{0} \quad \text{if } x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) \quad \text{if } x \neq 0 \\ 0 \quad \text{if } x = 0 \end{cases}$$

Does $f'(0)$ exist? $g'(0)$ exist? why?

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \frac{\cancel{h} \sin\left(\frac{1}{h}\right)}{\cancel{h}} = \lim_{h \to 0} \sin\left(\frac{1}{h}\right) \quad \boxed{\text{DNE}}$$

$$g'(0) = \lim_{h \to 0} \frac{g(h)}{h}$$

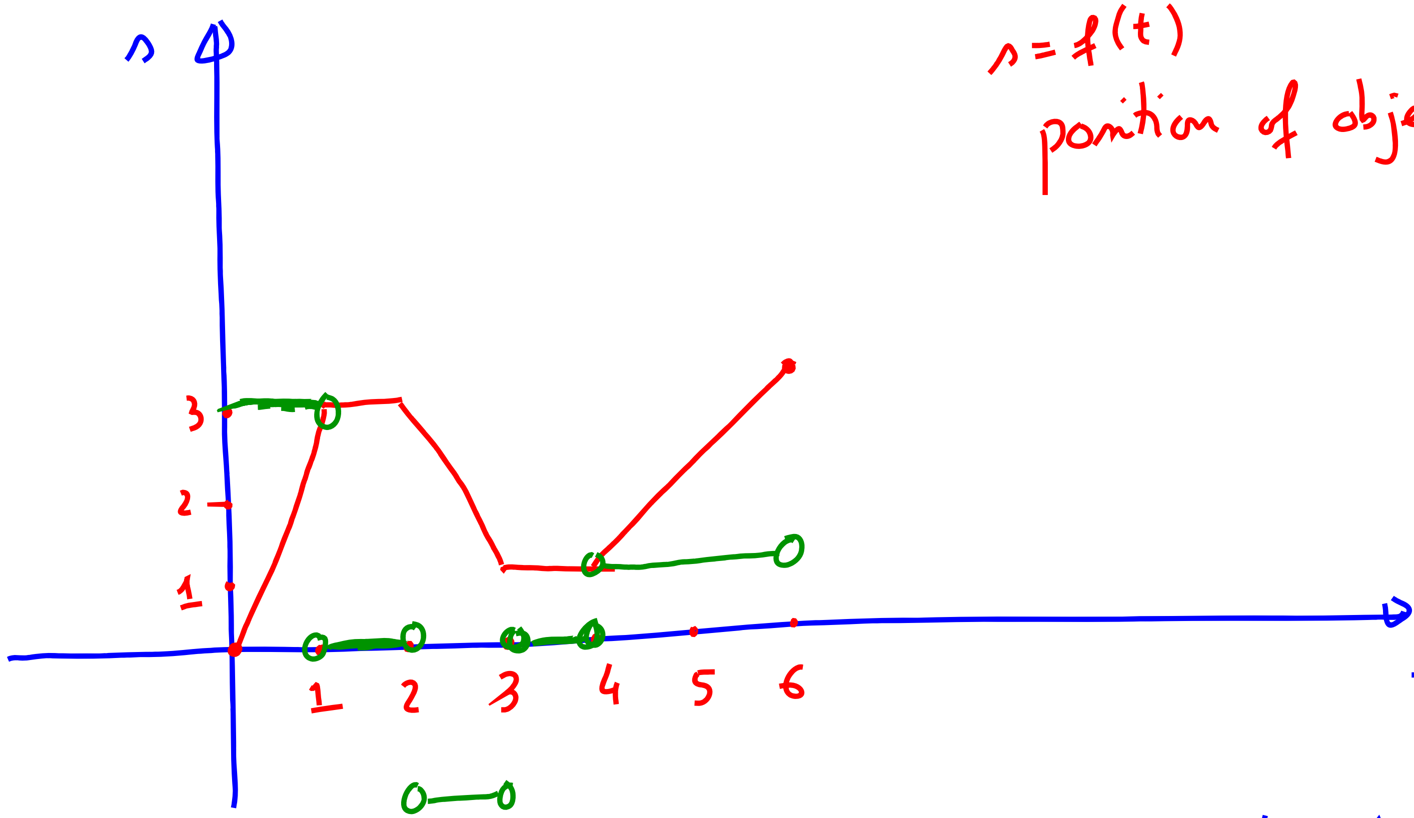
$$= \lim_{h \to 0} \frac{h^2 \cdot \sin\left(\frac{1}{h}\right)}{h} = \lim_{h \to 0} h \cdot \sin\left(\frac{1}{h}\right)$$

$$-1 \leq \sin\left(\frac{1}{h}\right) \leq 1$$

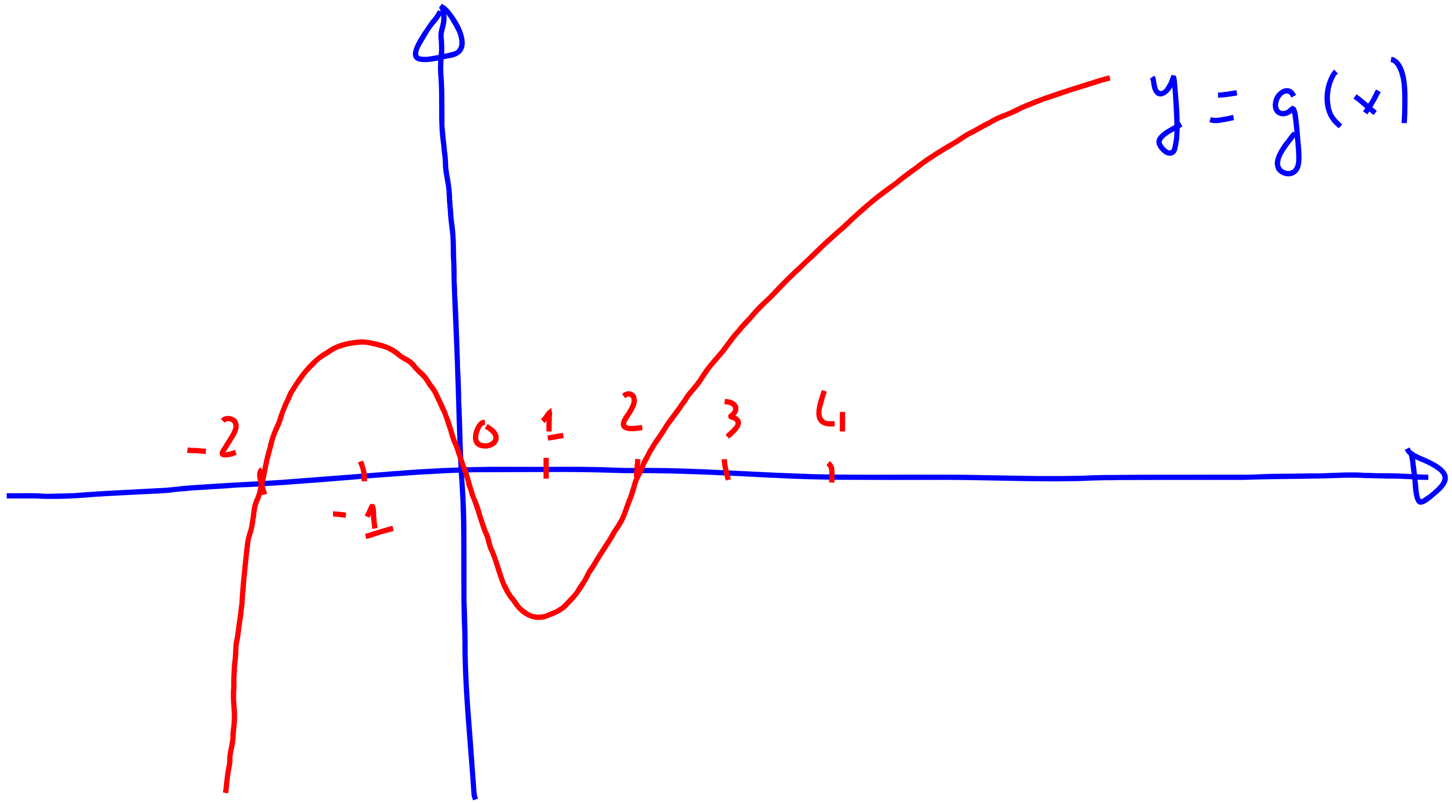
$$-h \leq h \sin\left(\frac{1}{h}\right) \leq h \; ; \quad -h \geq h \sin\left(\frac{1}{h}\right) \geq h$$

$$0 \qquad 0 \qquad 0 \qquad\qquad 0 \qquad 0 \qquad 0$$

$s = f(t)$
position of object at t.



Plot the graph of the velocity of the object with respect to time



Arrange the #'s $g'(-2)$; $g'(0)$; $g'(2)$; $g'(4)$; 0.

in increasing order.

The derivative as a function.

The formula $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

gives us the derivative of $y = f(x)$ at a specific point $x = a$.

In many situations, it will be more useful to have a formula for the derivative at any arbitrary point x. We just need to replace a by x in the first formula:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

(provided limit exists)

This defines a function of x. That function $y = f'(x)$ is called the derivative function of f. In short, the derivative.

Domain of $y = f'(x) = \{ x \mid f'(x) \text{ exists} \}$

E.g. $f(x) = x^3$.

Find a formula for the derivative $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \to 0} (3x^2 + 3x\cancel{h}^{0} + \cancel{h^2}^{0})$$

$$\boxed{f'(x) = \boxed{3x^2}}$$

$f'(-1) = 3$; $f'(\frac{1}{2}) = \frac{3}{4}$

$f'(-2) = 12$

$f'(0) = 0$

Def: We say that f is <u>differentiable</u> at a point x if $f'(x)$ exists.

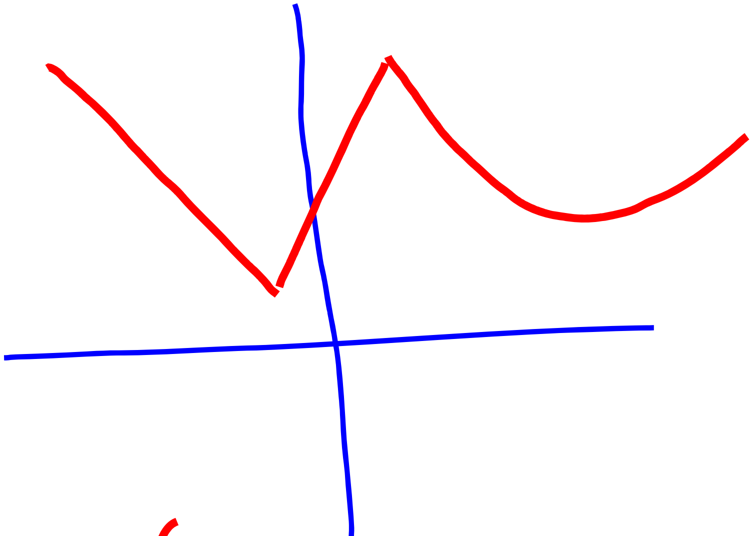
E.g. $f(x) = x^3$; $f'(x) = 3x^2$ So this function is differentiable everywhere (in $(-\infty, \infty)$)

$f(x) = \frac{1}{x}$; $f'(x) = -\frac{1}{x^2}$. So f is differentiable at any number except for 0 (in $(-\infty, 0) \cup (0, \infty)$)

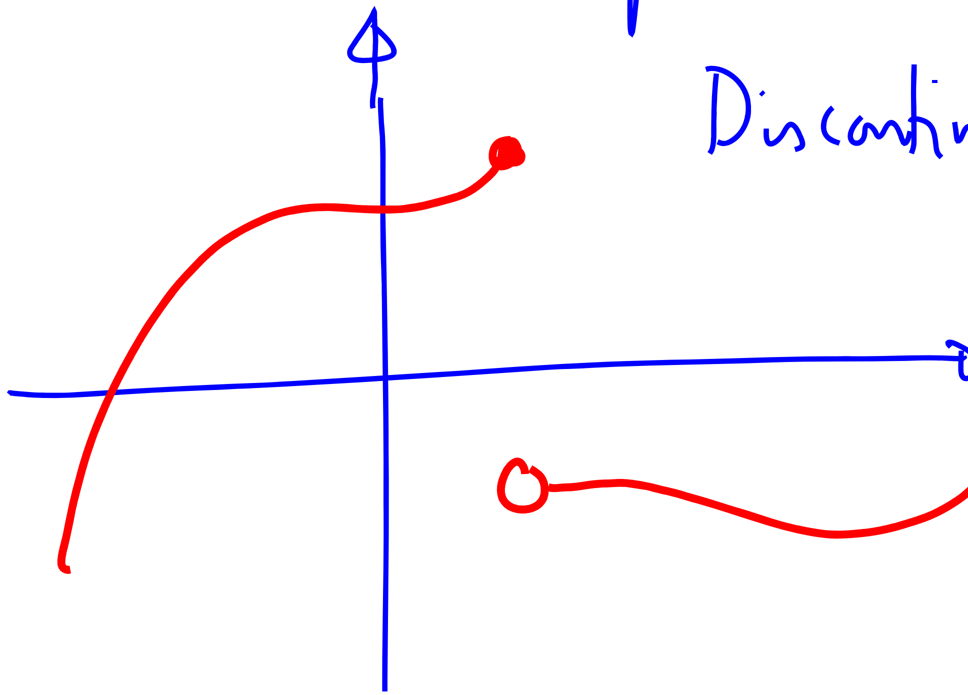
Ex: Prove that $f(x) = |x|$ is differentiable everywhere except for $x = 0$. Hint: $|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$

$\bigvee$

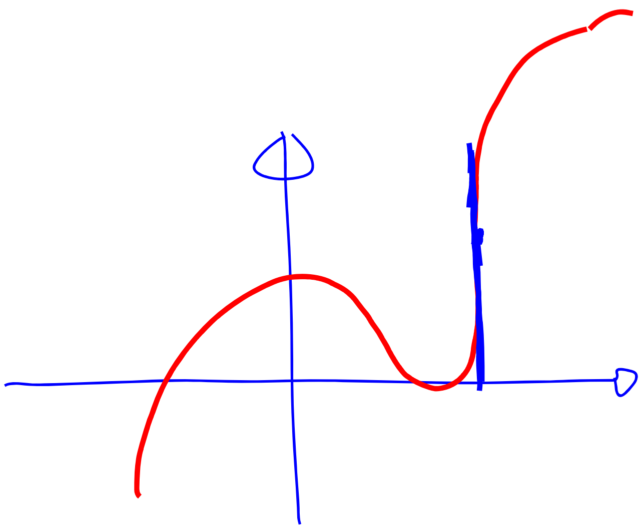
In general, if a graph has a sharp corner at a pt, then the derivative DNE at that point.



Corners



Discontinuity.



vertical tangent line

Differentiability is a stronger notion than continuity.

<u>Theorem</u>: If f is differentiable at a point, then f will be continuous at that point.