3.5. Derivative of Trig Functions

$f(x) = \sin x$.

Then $f'(x) = \cos x$.

In Leibnitz notation:

$$\frac{d}{dx}[\sin x] = \cos x$$

E.g. Find $\frac{dy}{dx}$ given $y = x^2 \cdot \sin(x)$

$$\frac{dy}{dx} = 2x\sin(x) + x^2\cos(x) \quad \text{(Product Rule)}$$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

$$\frac{d}{dx}[\cos x] = -\sin x.$$

$$\boxed{\frac{d}{dx}\left[\tan x\right] = \sec^2 x.}$$

Prf: $\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}\left[\cot x\right] = -\csc^2 x. \quad ; \quad \frac{d}{dx}\left[\csc x\right] = -\csc x \cot x.$$

$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$$

$$\frac{d}{dx}(\sin x) = \cos x \quad ; \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad ; \quad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \quad ; \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Higher derivatives $y = \sin x$; $y = \cos x$

$y = \sin x$

$\frac{dy}{dx} = \cos x$; $\frac{d^2y}{dx^2} = -\sin x$; $\frac{d^3y}{dx^3} = -\cos x$; $\frac{d^4y}{dx^4} = \sin x$

What is $\frac{d^{2017}y}{dx^{2017}}(\sin x) = \cos x$

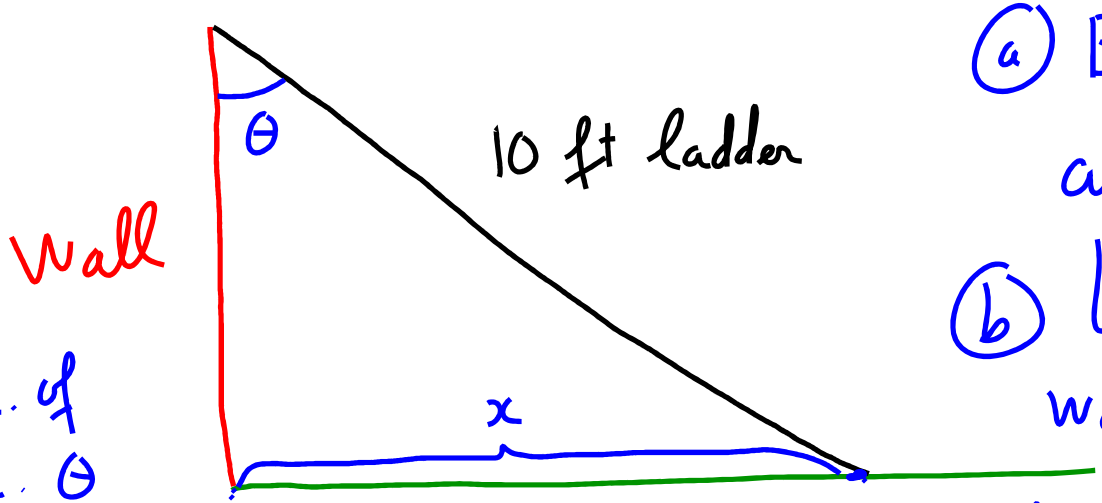
In general, $\frac{d^n y}{dx^n}(\sin x)$

- $\cos x$; $4 \mid n$ $R = 1$
- $-\sin x$; $4 \mid n$ $R = 2$
- $-\cos x$; $4 \mid n$ $R = 3$
- $\sin x$; $4 \mid n$; $R = 0$

$\frac{d^n y}{dx^n}(\cos x) =$

$$\frac{d^n}{dx^n}(\cos x) = \begin{cases} -\sin x & \text{if } 4 \mid n \quad R = 1 \\ -\cos x & \text{if } 4 \mid n \quad R = 2 \\ \sin x & \text{if } 4 \mid n \quad R = 3 \\ \cos x & \text{if } 4 \mid n \quad R = 0. \end{cases}$$

E.g.



(a) Express the distance x as a function of θ

(b) ladder slides away from wall, how fast does x change w.r.t. θ when $\theta = \frac{\pi}{3}$.

R.O.C. of x w.r.t. θ when $\theta = \pi/3$