

Integrals that involve Exponential and Logarithmic Function

Recall:

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (a^x) = a^x \cdot \ln a$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

E.g.

$$-\int e^{-x} (-dx) = -\int e^u du = -e^u + C = -e^{-x} + C$$

$$u = -x ; \quad du = -dx$$

E.g. $\int e^x \cdot \sqrt{1 + e^x} dx$ $\rightarrow du$

$$\text{let } u = 1 + e^x \quad du = e^x dx$$

$$\int \sqrt{u} du = \int u^{1/2} du = \frac{2u^{3/2}}{3} + C = \frac{2}{3} (1 + e^x)^{3/2} + C$$

E.g. $\int_1^2 \frac{e^{1/x}}{x^2} dx$

$$\text{let } u = e^{1/x} \quad du = e^{1/x} \cdot \left(-\frac{1}{x^2}\right) dx$$

$$\begin{aligned} \int (-du) &= \int (-du) = -u = -e^{1/x} \Big|_1^2 \\ &= -e^{1/2} + e^1 = e - \sqrt{e} \end{aligned}$$

1, 2, 8, 9, 10, 15.

Integrals that involve logarithmic Functions

Recall: $\frac{d}{dx} (\ln|x|) = \frac{1}{x}$

$$\frac{d}{dx} (\log_a |x|) = \frac{1}{x \ln a}$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dx}{x \ln a} = \log_a |x| + C \quad \left| \quad \frac{1}{\ln a} \int \frac{dx}{x} = \frac{1}{\ln a} \cdot \ln|x| + C \right.$$

$$\int \frac{1}{u} du = \ln|u| + C.$$

E.g.

$$- \int_0^{\pi/2} \frac{-\sin x}{1 + \cos x} dx$$

$$\text{let } u = 1 + \cos x$$

$$du = -\sin x dx$$

$$- \int \frac{du}{u} = -\ln|u| = -\ln|1 + \cos x| \Big|_0^{\pi/2}$$

$$= -\ln\left|1 + \cos\left(\frac{\pi}{2}\right)\right| + \ln|1 + \cos(0)|$$

$$= -\ln 1 + \ln 2 = \boxed{\ln 2}.$$