5.7. Integrals that result in Inverse Trig Functions

Basic Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2 - 1}}$$

Basic Integrals that result in Inverse Trig Functions

1. $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

2. $$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

3. $$\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$$

① $\displaystyle\int \frac{dx}{25+4x^2}$

② $\displaystyle\int \frac{\cos^{-1}(6t)}{\sqrt{1-36t^2}}\,dt$

③ $\displaystyle\int \frac{dt}{t\cdot\sqrt{1-\ln^2(t)}}$

④ $\displaystyle I(A) = \int_{-A}^{A} \frac{dt}{1+t^2}$

$$\lim_{A\to\infty} I(A)$$

① $\displaystyle\int \frac{dx}{25+4x^2} = \frac{1}{2}\int \frac{2\,dx}{25+(2x)^2}$

$u = 2x \; ; \; du = 2\,dx$

$\displaystyle\frac{1}{2}\int \frac{du}{25+u^2} = \frac{1}{2}\cdot\frac{1}{5}\cdot\tan^{-1}\left(\frac{u}{5}\right)+C$

$\displaystyle = \frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right)+C.$

② $\displaystyle -\frac{1}{6}\int \frac{\cos^{-1}(6t)\,-6}{\sqrt{1-36t^2}}\,dt$ du

$u = \cos^{-1}(6t)$

$\displaystyle du = \frac{-6}{\sqrt{1-36t^2}}\,dt$

$\displaystyle = -\frac{1}{6}\int u\,du = -\frac{u^2}{12}+C = -\frac{(\cos^{-1}(6t))^2}{12}+C$

(3) $u = \ln t$. $du = \dfrac{dt}{t}$

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C = \sin^{-1}(\ln t) + C.$$

(4) $$\int_{-A}^{A} \frac{dt}{1+t^2} = \arctan(A) - \arctan(-A)$$

$$\lim_{A \to \infty} \Big(\underbrace{\arctan(A)}_{\frac{\pi}{2}} - \underbrace{\arctan(-A)}_{-\frac{\pi}{2}} \Big) = \pi.$$