4.6 - Limits at Infinity and Asymptotes

HW # 1:

dominating term

$$\lim_{x\to\infty} \frac{\boxed{5x^2 + 16x + 20017}}{4x^2 + 13 + 10x} \approx \lim_{x\to\infty} \frac{5\cancel{x^2}}{4\cancel{x^2}} = \lim_{x\to\infty} \frac{5}{4} = \boxed{\frac{5}{4}}$$

dominating term

behaves like when x is large

To make this a little bit more rigorous.

$$\lim_{x\to\infty} \frac{\dfrac{5x^2 + 16x + 20017}{x^2}}{\dfrac{4x^2 + 13 + 10x}{x^2}} = \lim_{x\to\infty} \frac{5 + \dfrac{16}{x} + \dfrac{20017}{x^2}}{4 + \dfrac{13}{x^2} + \dfrac{10}{x}} = \boxed{\frac{5}{4}}$$

$\dfrac{16}{x} \to 0$, $\dfrac{20017}{x^2} \to 0$, $\dfrac{13}{x^2} \to 0$, $\dfrac{10}{x} \to 0$

Note: Find $\lim_{x \to \infty} \frac{p(x)}{q(x)}$ where p and q are polymials.

* Degree $p(x)$ = Degree $q(x)$.

$$\lim_{x \to \infty} \frac{p(x)}{q(x)} = \frac{a_n}{b_n}$$ where a_n ; b_n are the leading coefficients of $p(x)$ and $q(x)$ respectively.

* Degree $p(x)$ < Degree of $q(x)$

$$\lim_{x \to \infty} \frac{p(x)}{q(x)} = 0$$

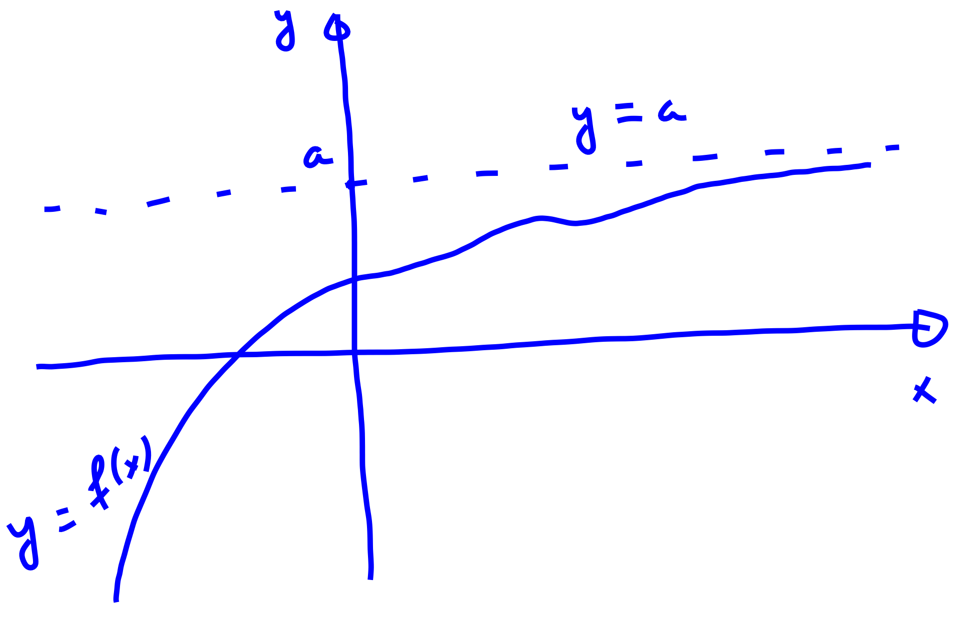
* Degree $p(x)$ > Degree of $q(x)$

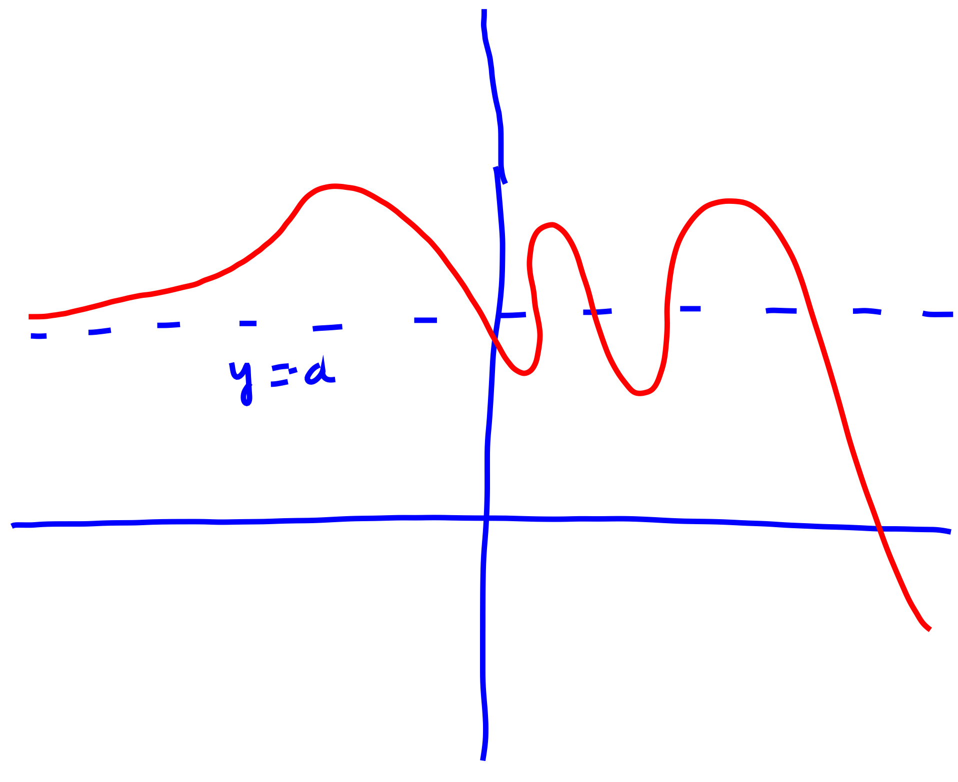
$\lim_{x \to \infty} \frac{p(x)}{q(x)} = \infty$ or $\lim_{x \to \infty} \frac{p(x)}{q(x)} = -\infty$ depending on the situation.

E.g. $\displaystyle\lim_{t\to-\infty}\frac{t}{\sqrt{4t^2+5}} \approx \frac{t}{\sqrt{4t^2}} \approx \frac{t}{2|t|}$

$$\lim_{t\to-\infty}\frac{t}{2|t|} = \lim_{t\to-\infty}\frac{t}{2(-t)} = \lim_{t\to-\infty}\left(-\frac{1}{2}\right)$$
$$= -\frac{1}{2}.$$

Horizontal Asymptotes





If $\lim_{x \to \infty} f(x) = a$ then $y = a$ is a H.A. of the function f

If $\lim_{x \to -\infty} f(x) = b$ then $y = b$ ——

(Note: a, b: finite #)

E.g. $f(x) = \dfrac{\sqrt{36x^2+7}}{8x+7}$

$$\lim_{x \to \infty} \frac{\sqrt{36x^2+7}}{8x+7} = \lim_{x \to \infty} \frac{6x}{8x} = \frac{3}{4}$$

$$\lim_{x \to -\infty} \frac{\sqrt{36x^2+7}}{8x+7} = \lim_{x \to -\infty} \frac{-6x}{8x} = -\frac{3}{4}$$

H.A.:

$y = \frac{3}{4}$

$y = -\frac{3}{4}$.