Linear Approximations and Differentials

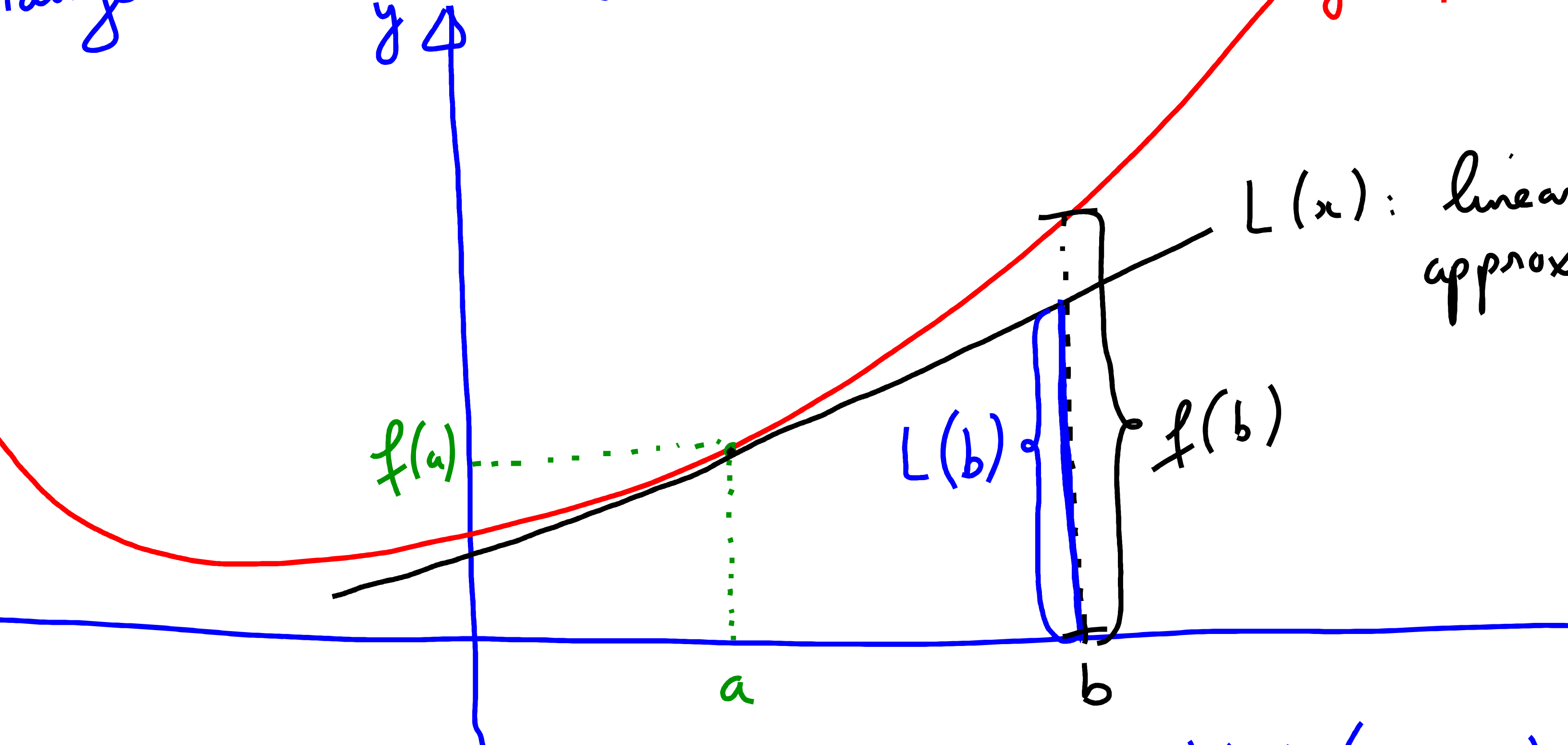
linear approximation of a function at a point.

on $y = f(x)$.

linear approximation of f at the point $(a, f(a))$ is t

n $L(x)$ whose equation is the point-slope equation

tangent line to the graph of f at $(a, f(a))$



pe of tangent line: $f'(a)$.

$$y - f(a) = f'(a)(x - a)$$

$$y = f(a) + f'(a)(x$$

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

$f(x) = \sqrt{x}$

nd the linear approximation of f at $a = 9$.

e this approximation to estimate $\sqrt{9.1}$

$$(x) = f(a) + f'(a)(x - a) \quad \begin{cases} f(x) = \sqrt{x} \\ a = 9 \end{cases}$$

$$) = \frac{1}{2\sqrt{x}} \;;\quad f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6} \;.\quad f(9) = 3$$

$$(x) = 3 + \frac{1}{6}(x - 9) = \boxed{\frac{1}{6}x + \frac{3}{2}}$$

stimate $\sqrt{9.1}$

$$L(9.1) = 3 + \frac{1}{6} \cdot (9.1 - 9) = 3 + \frac{0.1}{6} = 3.001667$$

Estimate $\boxed{e^{-0.015}}$ using a linear approximation to a

unction $y = f(x)$ at a point $x = a$.

Same question for $\boxed{(1.99)^3}$.

$= e^x$. $a = 0$; $f(0) = 1$; $f'(x) = e^x$; $f'(0) =$

$) = 1 + 1 \cdot (x - 0) = \boxed{1 + x}$

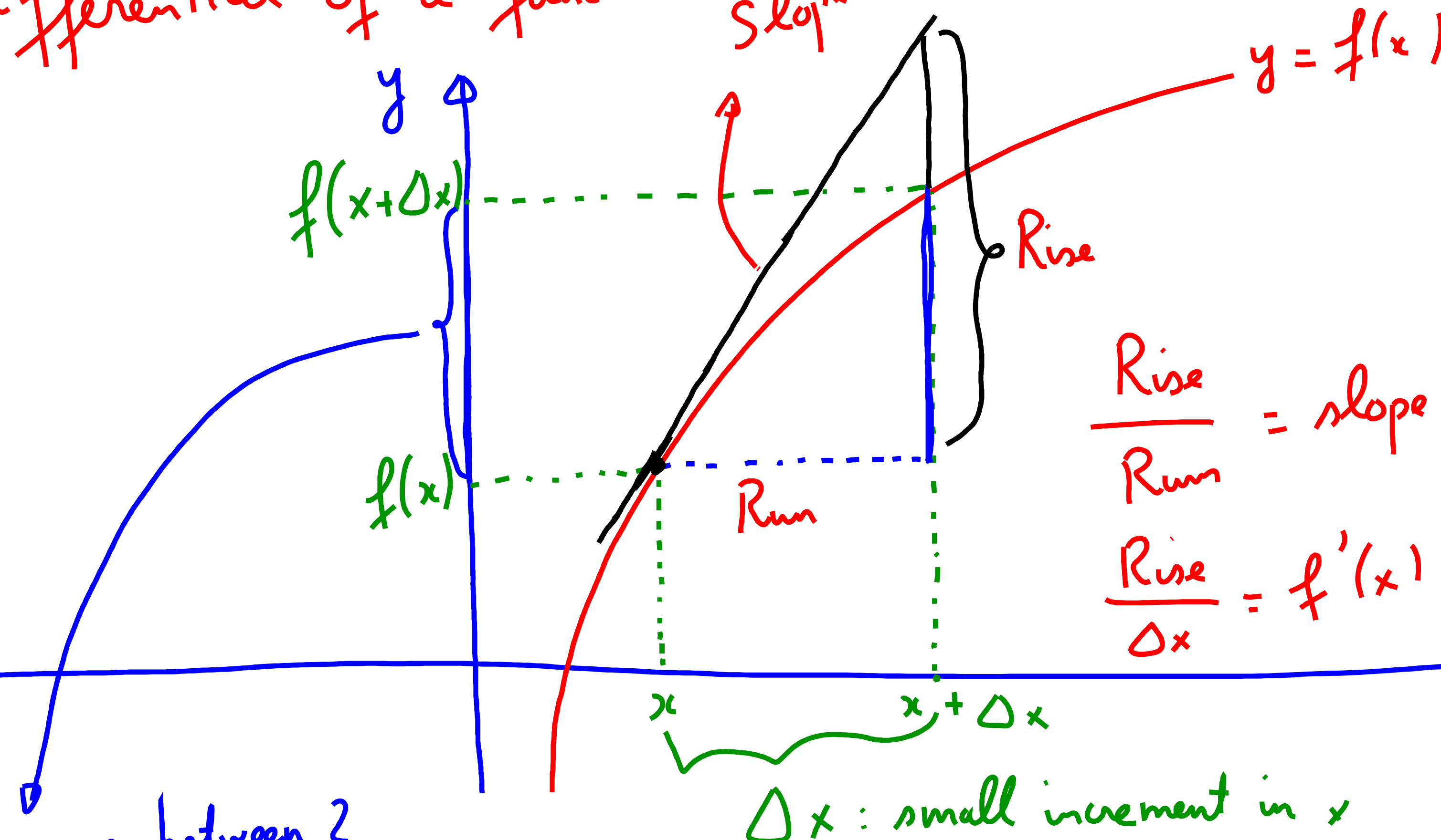
mate: $e^{-0.015}$: $1 + (-0.015) = 0.985$

$= x^3$. $a = 2$ $f(2) = 8$; $f'(x) = 3x^2$; $f'($

$) = 8 + 12(x - 2) = 12x - 16$

mate $(1.99)^3$: $L(1.99) = 8 + 12 \cdot \overbrace{(1.99 - 2)}^{-0.01} = \overbrace{8 - 0.}^{7.8}$

...fferential of a function.



$$\frac{\text{Rise}}{\text{Run}} = \text{slope} = f$$

$$\frac{\text{Rise}}{\Delta x} = f'(x)$$

...erence between 2 outputs

...e in $y = \Delta y$

$$\boxed{f(x+\Delta x) - f(x)}$$

$$\text{Rise} = f'(x)\,\Delta x$$

called the differential, the differen...

is denoted by dy.

$$\boxed{dy = f'(x)\,dx}$$

...e: $dx = \Delta x$

$dy \neq \Delta y$.

dy : differential

Δy : actual change in output $\Big\}$ dy...

$y = f(x) = x^3 + x^2 - 2x + 1$.

nd the actual change in the output of this function; i.

Δy when x changes from $\underline{2}$ to 2.05.

d the differential dy when x changes from 2 to 2

$= f(x + \Delta x) - f(x) = f(2.05) - f(2)$

$= \left((2.05)^3 + (2.05)^2 - 2\cdot(2.05) + 1\right) - \left(2^3 + 2^2 - \ldots\right.$

$= \boxed{0.717625}$ Actual change in output Δy

$dy = f'(x)\, dx$; $dx = \Delta x = 2.05 - 2 = 0.0\ldots$

$f'(x) = 3x^2 + 2x - 2$.

$dy = f'(2)\cdot(0.05) = \left(3\cdot(2)^2 + 2\cdot 2 - 2\right)\cdot(0.05)$

$= 14\cdot(0.05) = \boxed{0.7}$ dy :

Approximate error

Suppose the side length of a cube is measured to be $\boxed{5}$ with an accuracy of 0.1 cm

Use differential to estimate the error in the computed volume of the cube.

$\underline{x = 5}$; $\quad -0.1 \leq dx \leq 0.1$

$\boxed{dV} \approx \Delta V$

$V = x^3$; $\quad dV = 3x^2\, dx$

Approxiate ΔV ?

$-0.1 \leq dx \leq 0.1$

$\boxed{125}$ actual volume is in $\boxed{-7.5} \leq \underbrace{\boxed{75\, dx}}_{dV} \leq \boxed{7.5}$

$125 - 7.5 \leq \partial \leq 125 + 7.5$