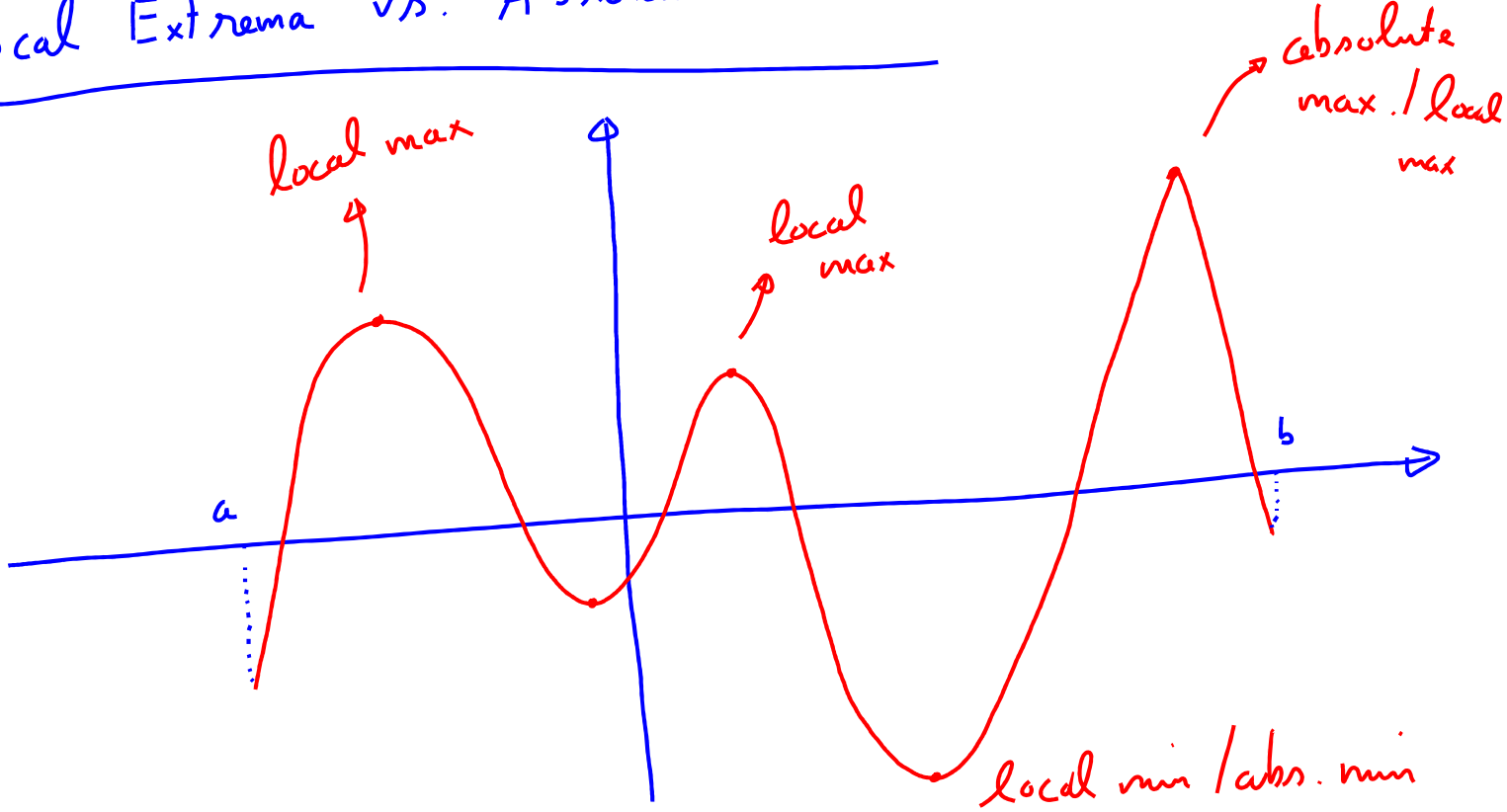
4.3. Finding Maxima and Minima of a function

Local Extrema vs. Absolute Extrema



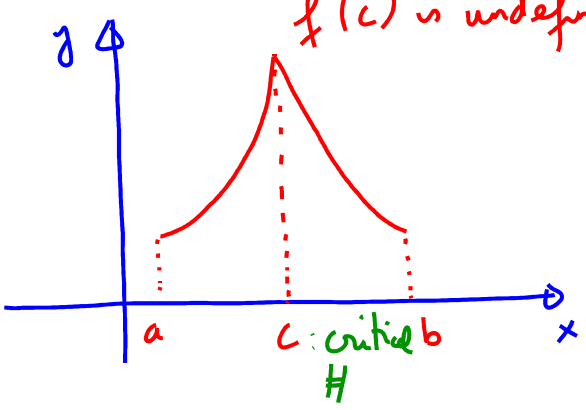
(1) Find absolute max / absolute min of a function over a close interval $[a,b]$

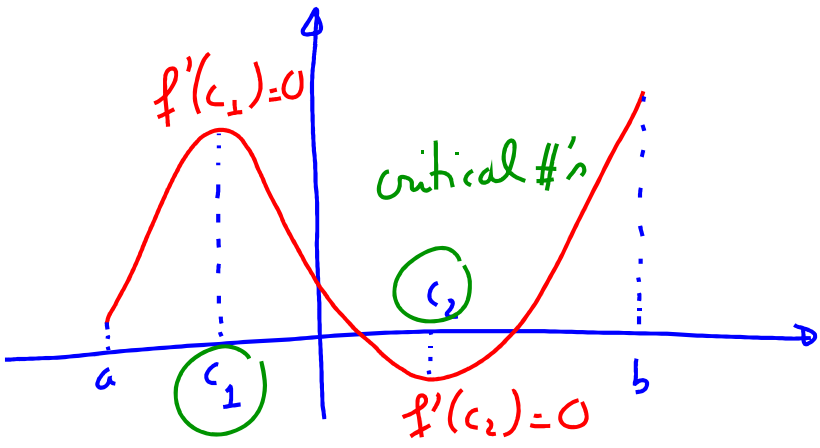
E.g. $f(x) = x^3 - 6x^2 + 9x + 1$ over the close interval $[0,5]$.

Find absolute max / min of f over this interval

Critical Points (critical numbers) of a function.

Def of a critical number: Let c be a number in the domain of f, c is called a critical number of f if $f'(c) = 0$ or $f'(c)$ is undefined.





E.g. $f(x) = x^3 - 6x^2 + 9x + 1$.

Find the critical numbers of f.

$$f'(x) = 3x^2 - 12x + 9$$

* $f'(x)$ is never undefined.
* $f'(x) = 0$ $\quad 3x^2 - 12x + 9 = 0$

$$3(x-1)(x-3) = 0$$

$$x = 1 \; ; \; x = 3.$$

* Critical numbers of f are $x = 1$; $x = 3$.

Ex. Find the critical points of the given function.

(a) $f(x) = \dfrac{4x}{1+x^2}$

(b) $g(x) = 4\sqrt{x} - x^2$.

(c) $h(x) = x^{\frac{3}{5}}(4-x)$

(d) Could you come up with a formula for a function with no critical points?

(a) $f'(x) = \dfrac{4-4x^2}{(1+x^2)^2}$

When is f' undefined? Never

When is $f' = 0$? $x = \pm 1$.

Critical points of f: $x = 1$; $x = -1$

(b) $g'(x) = \dfrac{2}{\sqrt{x}} - 2x$.

g' is undefined when $x = 0$

$g' = 0$ when $x = 1$

$\Big\}$ $x = 0$, $x = 1$: critical pts

(c) $h'(x) = \dfrac{12}{5}x^{-2/5} - \dfrac{8}{5}x^{3/5}$

$= \dfrac{4}{5}x^{-2/5}(3-2x)$

h' is undefined when $x = 0$

$h' = 0$ when $x = \dfrac{3}{2}$

$\Big\}$ $x = 0$, $x = \dfrac{3}{2}$: critical pts