Last time

c is a critical point of $f(x)$ if $f'(c) = 0$ or
$f'(c)$ is undefined.

If f is continuous on a closed interval $[a,b]$, then the maximum and minimum of f can only occur at the critical points or at the endpoints.

→ Closed interval method.

To find the maximum and minimum of a function f on a closed interval $[a,b]$

1. Find all the critical points of f within $[a,b]$
2. Evaluate f at these critical points in 1
3. Evaluate f at the endpoints $x = a$, $x = b$.
4. The largest value in 2 and 3 is the max of f.
   The smallest value in 2 and 3 is the min of f.

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0,5]$.

Find max / min of f on $[0,5]$.

(1) Find critical points in $[0,5]$.

$f'(x) = 3x^2 - 12x + 9$

* f' is defined everywhere
* $f' = 0$: $3x^2 - 12x + 9 = 0$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$\boxed{x=1}$; $\boxed{x=3}$ are critical points in

(2) Find $f(1)$; $f(3)$

$\boxed{f(1) = 5}$

$f(3) = (3)^3 - 6(3)^2 + 9(3) + 1$

$f(3) = 27 - 54 + 27 + 1 = \boxed{1}$

(3) Find $f(0)$; $f(5)$

$f(0) = \boxed{1}$; $f(5) = (5)^3 - 6(5)^2 + 9(5) + 1$

$= \boxed{21}$

(4) Conclusion: Abs. max of $f = 21$. it occurs at $x = 5$

Abs. min of $f = 1$. It occurs at $x = 0$ and $x = 3$.

Ex.

$$g(x) = x \cdot e^{-x^2/8}.$$

Find abs max / min of g on $[-1, 4]$.

① Critical Points

$$g'(x) = e^{-x^2/8} + x \cdot e^{-x^2/8} \cdot \left(\frac{-x}{4}\right)$$

$$= e^{-x^2/8} - \frac{x^2}{4} \cdot e^{-x^2/8}$$

* g' is defined everywhere.
* $g' = 0$.

$$e^{-x^2/8} - \frac{x^2}{4} \cdot e^{-x^2/8} = 0$$

$$\underbrace{e^{-x^2/8}}_{>0} \cdot \underbrace{\left(1 - \frac{x^2}{4}\right)} = 0$$

$$\Rightarrow 1 - \frac{x^2}{4} = 0 \Rightarrow \frac{x^2}{4} = 1 \Rightarrow x^2 = 4$$

$\Rightarrow x = \pm 2$. $x = -2$ does not belong $[-1, 4]$.

$$\boxed{x = 2}$$

② Find $g(2)$. $g(2) = 2 \cdot e^{-4/8} = 2e^{-1/2} \approx 1.21$

③ Find $g(-1)$; $g(4)$. $g(-1) = -e^{-1/8}$

$g(4) = 4e^{-2} \approx 0.54$

④ Abs min $= -e^{-1/8}$. occurs @ $x = -1$

Abs max $= 2e^{-1/2}$. occurs @ $x = 2$.

#6.

$$f'(x) = 2\cos x - 2\sin x$$

$$f' = 0 : \quad 2\cos x - 2\sin x = 0 \quad ; \quad \text{over } \underline{\left[0, \frac{\pi}{2}\right]}$$

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}$$

$$f(0) = \boxed{2}$$

$$f\left(\frac{\pi}{2}\right) = \boxed{2}$$

min

$$f\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{4} + 2\cos\frac{\pi}{4}.$$

$$= \sqrt{2} + \sqrt{2} = \boxed{2\sqrt{2}} \text{ max}$$