11.4 Bernoulli Trials and Binomial Distribution

Goals: (1) Understand Bernoulli trials and the binomial distribution.

(2) Solve some applications.

A Bernoulli trial is a sequence of experiments such that the following criteria are satisfied

(1) For each experiment in the sequence, there are only 2 possible outcomes

(2) Each outcome has a fixed probability of occurring

(3) Each experiment is completely independent of the others

Eg. Manager of a store.

Probability that a random customer will buy at least one product in the store is 0.3.

If 3 customers walk in the store, find probability that exactly 2 out of these 3 customers will buy something?

Use b event that a customer buy something

b' ——————— doesn't buy anything

Exactly 2 of 3 customers buy something:

bbb' ; bb'b ; b'bb

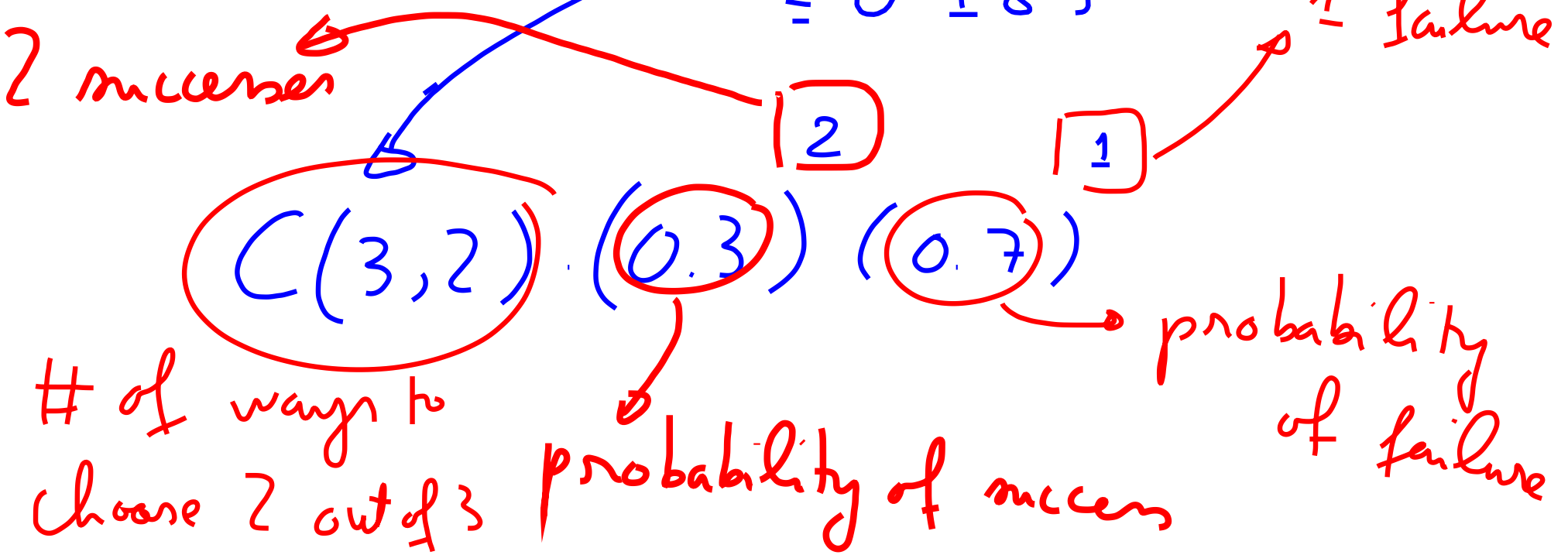
$P(bbb') = (0.3)(0.3)(0.7)$

$P(bb'b) = (0.3)(0.7)(0.3)$

$P(b'bb) = (0.7)(0.3)(0.3)$

$P(\text{exactly 2 successes}) = 3 \cdot (0.3)(0.3)(0.7)$

$= 0.189$



In general, if we have n customers, and the probability of success is p, then the probability that exactly x customers will buy something is

$$p(x) = C(n, x) \cdot p^{x} \cdot (1-p)^{n-x}$$

E.g. 20 True - False questions.

Probability to get 18 questions right if you guess randomly?

$$p(18) = C(20,18)\cdot(0.5)^{18}\cdot(0.5)^{2}$$

$$= 0.000181198$$

E.g. 20 questions. Each question has 4 choices. (only 1 correct choice)

Find the probability of getting 12 questions correctly if guess randomly

$$p(12) = C(20, 12) \cdot (0.25)^{12} \cdot (0.75)^{8}$$

$$= 0.000752$$

binomial distribution $(20, 0.25, 12)$

E.g. Suppose 15% of major league baseball players are left-handed. In a sample of 12 random players, find the probability that at most 6 of them are left handed.

$P(X \leq 6)$: $P(0) + P(1) + P(2) +$

$P(3) + P(4) +$

$P(5) + P(6)$

at most 6 are left-handed

$= 0.999$

The mean of a binomial distribution where the # of trials is n and the probability of each success is p is $\boxed{np}$

The standard deviation is $\sqrt{np(1-p)}$.

E.g. Toss 100 coins. Success = get a H

$$\text{Mean} = np = 100 \cdot (0.5) = 50$$

$$\text{S.D.} = \sqrt{np(1-p)} = \sqrt{100 \cdot (0.5)(0.5)}$$

$$= \sqrt{25} = 5$$