# 1.1.5. Normal Distribution

Goals: (1) Normal Distribution

(2) Area under the normal curve

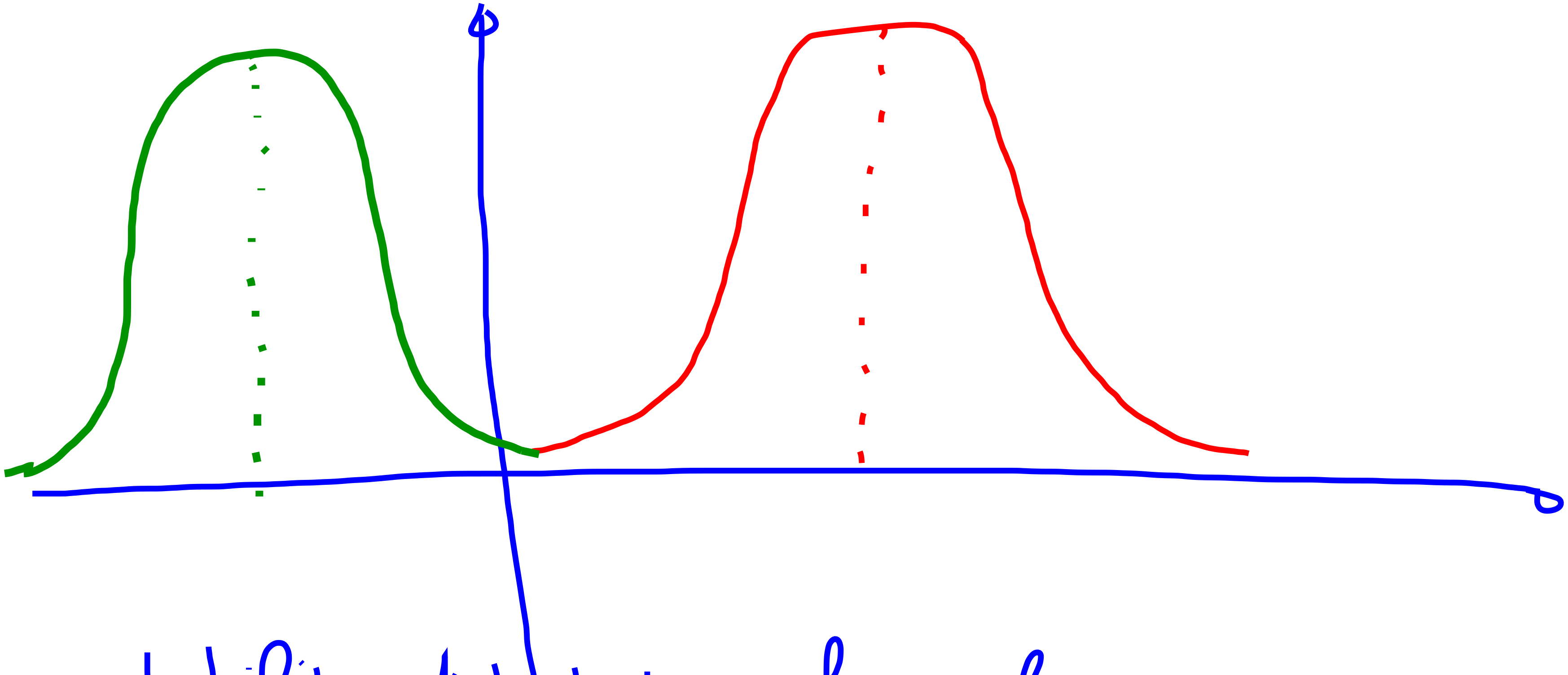
(3) Approximate Binomial Distribution by a normal distribution.

So far, we deal with discrete random variable

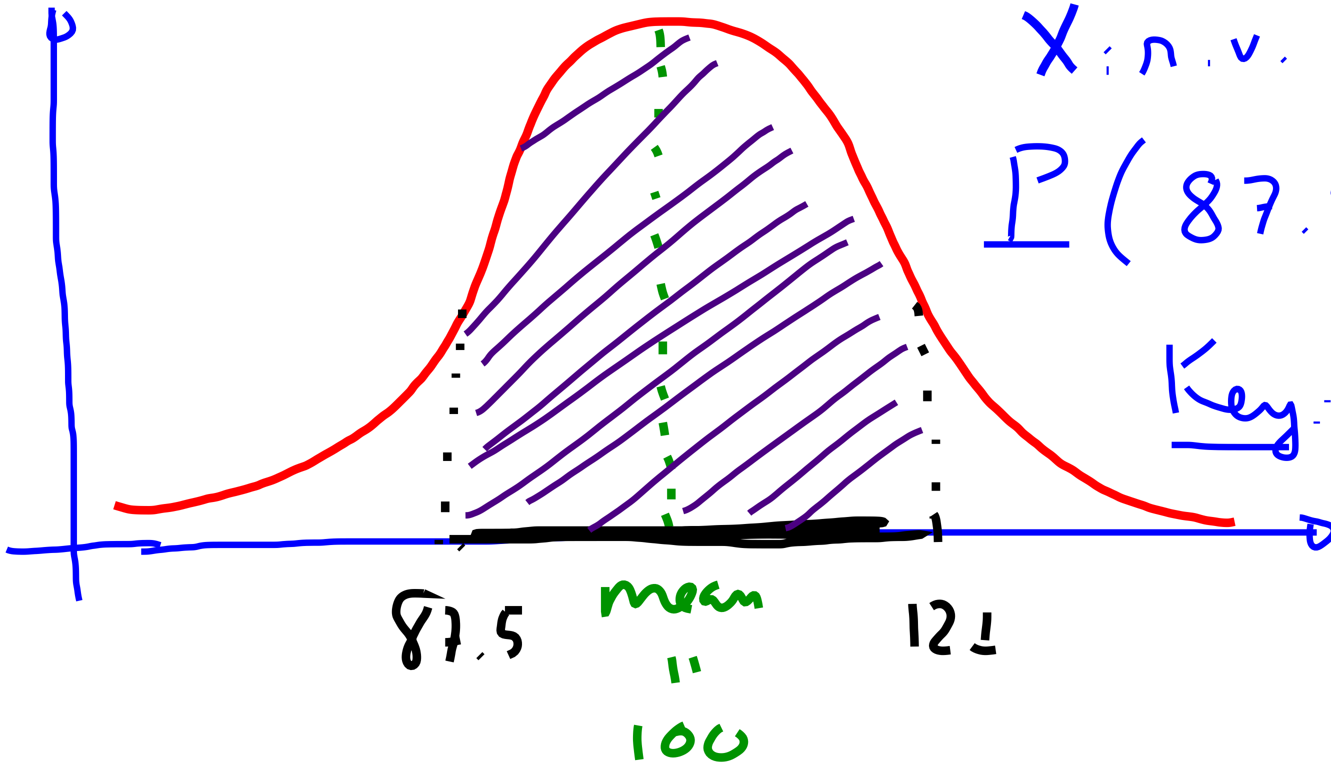
Continuous R.V.: heights of people

life expectancy of light bulbs

For many continuous r.v., most of the data points fall near the center. fewer data fall near the end



The probability distribution of such r.v. is called the normal distribution. The graph is called a normal curve or bell curve




$X$ : r.v.

$P(87.5 \le X \le 121)$

Key: Probability = area under the normal curve

$P(a \le X \le b)$ = area under the normal curve from $a$ to $b$

→ Use z-score table to find that area.

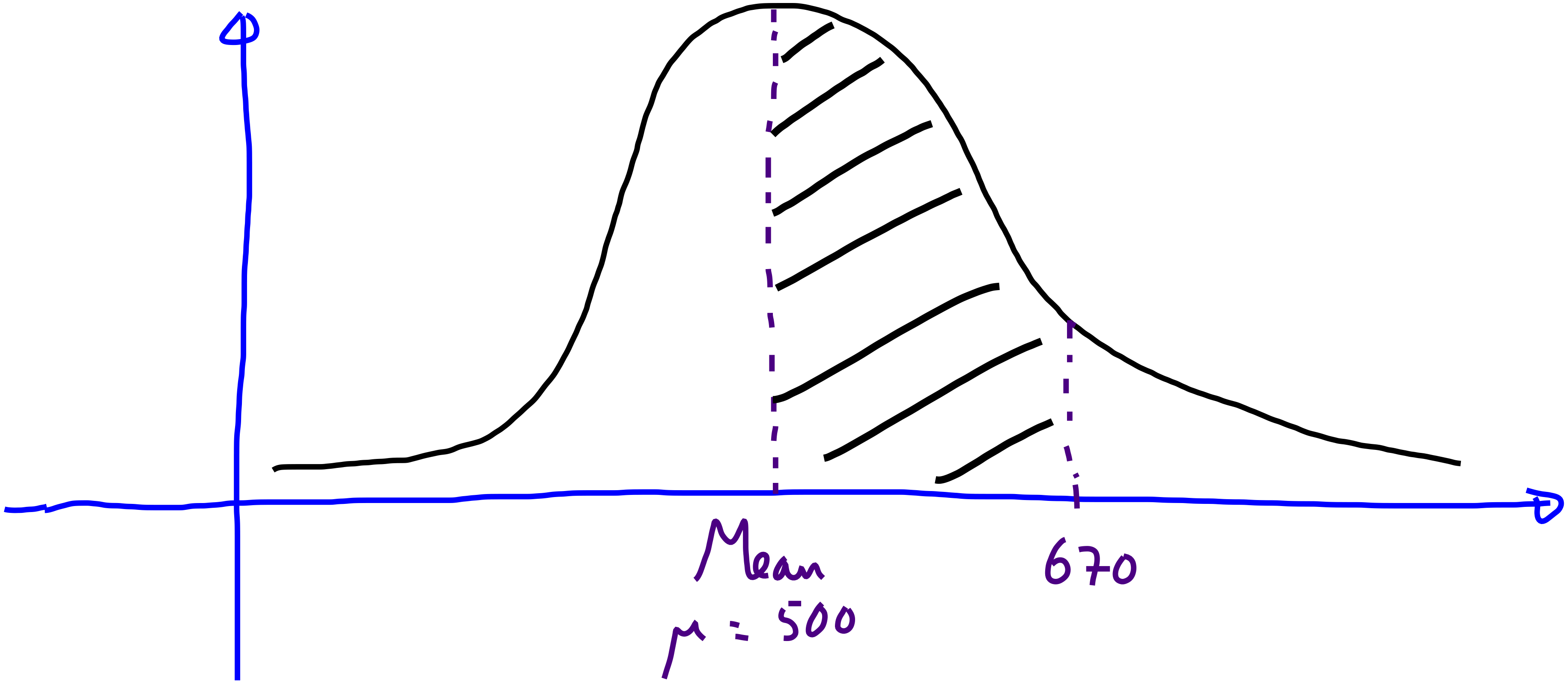
E.g. Manufacturer produces light bulbs.

Life expectancy of these light bulbs follow a normal distribution with mean = 500 hours ($\mu$) and standard deviation = 100 hours ($\sigma$)

What percentage of light bulbs can be expected to last between 500 and 670 hours?

R.V. X = life expectancy of a randomly chosen light bulb

→ Find $P(500 \leq X \leq 670) = ?$




$P(500 \le X \le 670)$ = area under normal curve from 500 to 670

z - score table give you the area under the curve from the mean to a point that is z - standard deviation from the mean

To find $P(500 \leq X \leq 670)$

we need to find how many s.d. is 670

from the mean?

$$z\text{-score} = \frac{670 - 500}{100} = \frac{170}{100}$$

(for 670)

$$= 1.7$$

From z-score table: area under normal

curve from 500 to 670 is 0.4554

Hence, $P(500 \leq X \leq 670) = 0.4554$

$\longrightarrow$ 45.54% of

light bulbs will have life expectancy from 500 hours to 670 hours

$$P(X \geq 670) = 0.5 - 0.4554$$

$$\approx 0.0446$$

$$\longrightarrow 4.46\%$$

$$P(380 \leq X \leq 500) = ?$$

$$z\text{-score} = \frac{380 - 500}{100} = -1.2 \quad 0.3849$$

look up z-score $= 1.2$ (by symmetry) $\rightarrow$

In general,

to find $z$-score of $x$-value $= \dfrac{x - \mu}{\sigma}$.

Ex. Company sells dog food.

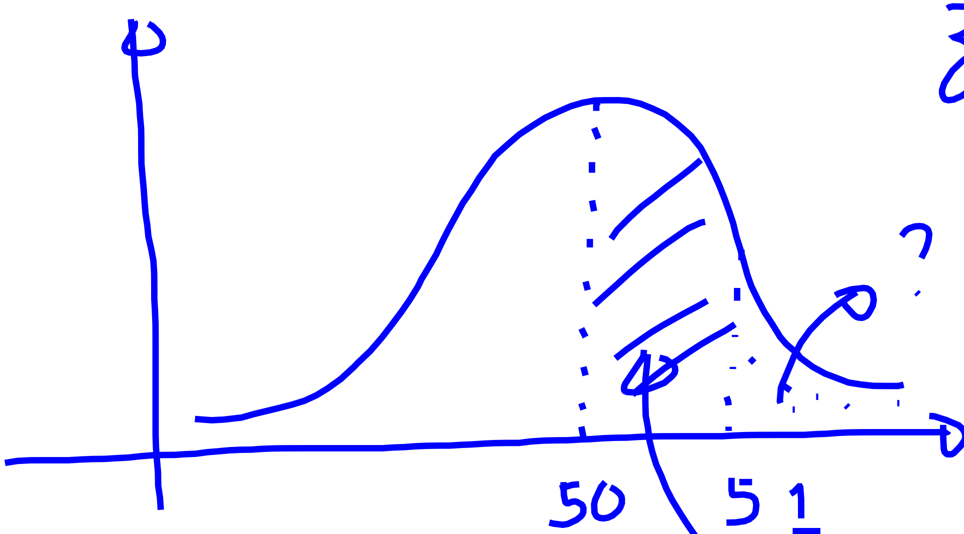
Weights of dog food bags are normally distributed with mean $(\mu) = 50$ lbs and S.D. $\sigma = 0.5$ lbs.

(a) Find the probability that a randomly chosen bag will weigh more than 51 lbs

(b) less than 49 lbs

(c) From 49 to 51 lbs

(a) $P(X \geq 51) = ?$




z-score of 51:

$$\frac{51-50}{0.5} = 2.$$

z-score table:

area from 50 to 51 is

0.4772

$$P(X \geq 51) = 0.5 - 0.4772 = 0.0228$$

(b) $P(X \leq 49)$

By symmetry, $P(X \leq 49) = 0.0228$

(c) $P(49 \leq X \leq 51)$

$= 2 \cdot (0.4772) = 0.9544.$

Use the normal distribution to approximate binomial distribution.

E.g. Credit card company claims. 40% of their customers use the card to buy gas

Binomial distribution.

Customer
- use card : $p = 0.4$
- doesn't use card : $q = 0.6$

Select a sample of 20 customers.

(a) Probability that 6 to 12 of them use the card. $P(6 \le X \le 12) = ?$

(b) Probability that less than 4 customers use the card. $P(X \le 4)$.

Use normal distribution to approximate this

Process: 1 Find mean and S.D. of the binomal distribution.

$$\mu = n \cdot p = 20 \cdot 0.4 = 8$$

n : sample size (# of experiments)
p : probability of success

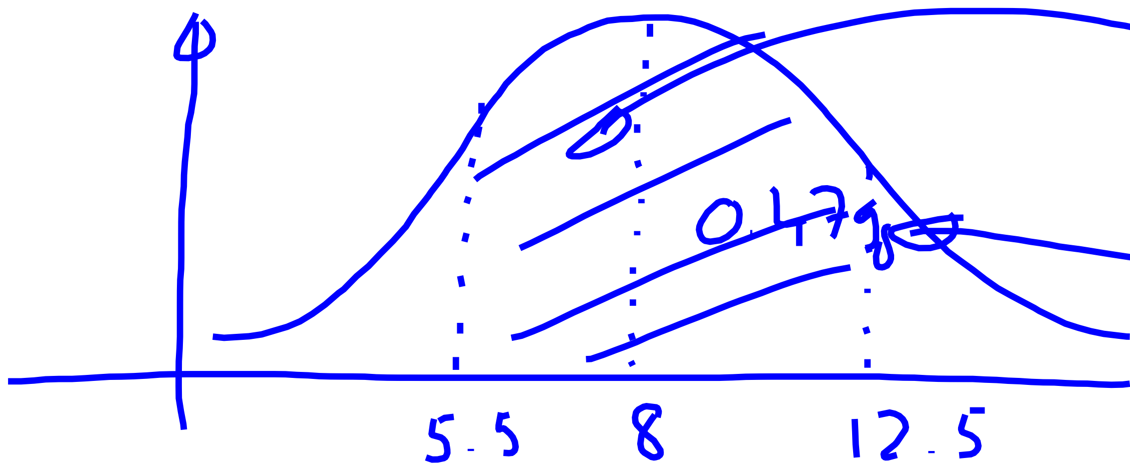
$$S.D.\ \sigma = \sqrt{npq} = \sqrt{20 \cdot (0.4)(0.6)}$$
$$= 2.19$$

Step 2 : Rule of 0.5 in using normal dist to approximate binomal dist.

$P(6 \le X \le 12) \rightarrow$ binomial

$6 - 0.5 = 5.5$ $\qquad$ $12 + 0.5 = 12.5$

$\longrightarrow P(5.5 \le X \le 12.5) \longrightarrow$ normal




z - score for 5.5 : $\dfrac{5.5 - 8}{2.19} = -1.14 \rightarrow 0.3729$

z - score for 12.5 : $\dfrac{12.5 - 8}{2.19} = 2.05 \rightarrow 0.4798$

$$P(5.5 \leq x \leq 12.5) = 0.3729 + 0.4798$$

$$= \ldots$$

(b) $P(X \leq 4) \rightarrow$ binomial.

$\xrightarrow[\text{0.5 rule}]{}$ $P(x \leq 4.5) = 1 - 0.4452$

$$= \ldots$$