3.2. Compound Interest

Goals: ① Understand and Apply the Compound Interest Formula.

② Understand and Apply the Annual percentage yield formula

Compound Interest

What is the difference between compound interest and Simple Interest

Eg. Put $100 into a bank account

Annual Interest Rate: 6%

Compound Monthly $\frac{6}{12} = 0.5\% = 0.005$

1st month: $\$100 + \$100 \cdot (0.005)$

$100(1 + 0.005)$

2nd month: $100(1 + 0.005) + 100(1+0.005)(0.005)$

$100(1+0.005)(1 + 0.005)$

$100(1+0.005)^2$ ⟵ 2nd month

$100(1+0.005)^3$ ⟵ 3rd month

⋮

$100(1+0.005)^{12}$ ⟵ 12th month

$\$106.7$

Compound Interest Formula

$$A = P\left(1 + \frac{R}{m}\right)^{mt}$$

A = future amount

P = principal

R = annual interest rate

m = # of compounding periods per year

t = # of years

Simplified formula:

$$i = \frac{R}{m} = \text{interest rate per compounding period}$$

$$n = mt = \text{total \# of compounding periods}$$

$$\boxed{A = P(1 + i)^n}$$

Eg 1 Bank pays 6% compounded semiannually.

You want to have $8000 after 4 years.

How much money should you deposit now?

$$A = P(1+i)^n$$

$A = \$8000$ $\quad n = 8$

$i = \frac{6\%}{2} = 3\% = 0.03$

$8000 = P(1+0.03)^8$

$8000 = P(1.03)^8$

$8000 = P \cdot 1.27$

$P = \frac{8000}{1.27}$

$P \approx 6299.21$

③ (Review of the natural logarithmic function)

ln : log base e

How long will it take for $2000 to grow to $22,000 if it is invested at 4% compounded monthly.

$A = P(1+i)^{n}$ $\quad i = \frac{4\%}{12} = 0.33\%$

$A = P(1+i)^{mt}$ $\quad = 0.0033$

$m = 12$.

$$22000 = 2000\left(1.0033\right)^{12t}$$

$$11 = \left(1.0033\right)^{12t}$$

Take the natural log: ln of both sides:

$$\ln(11) = \ln\left(1.0033\right)^{12t}$$

$$\ln(11) = (12t)\cdot \ln(1.0033)$$

$$t = \frac{\ln(11)}{12\ln(1.0033)} \approx 61 \text{ years}$$

Annual Percentage Yield (APY)

The annual percentage yield is the simple interest rate that will produce the same amount as a given compound interest rate in a year.

Eg. $100 invested at 6% compounded monthly for 1 year

APY ?

$100 + 100 · APY

100 (1 + APY)

$$100 \cdot \left(1 + \frac{0.06}{12}\right)^{12}$$

$$\cancel{100}(1 + APY) = \cancel{100}\left(1 + \frac{0.06}{12}\right)^{12}$$

$$1 + APY = \left(1 + \frac{0.06}{12}\right)^{12}$$

$$APY = \left(1 + \frac{0.06}{12}\right)^{12} - 1$$

$$APY = 1.06 - 1 = 0.06$$

$\longrightarrow 6\%$

Formula for APY

$$APY = \left(1 + \frac{R}{m}\right)^{m} - 1$$