

3.3. Future Value of an Annuity; Sinking Funds

- Goals:
- ① Compute the future value of an annuity
 - ② Solve problems involve sinking funds

What is an annuity?

An annuity is a sequence of equal periodic payments

E.g. Retirement Account
Contribution: \$1000 every 3 month (quarterly)
Company: \$1000

Total: \$2000 quarterly contributed to retirement account
Invest this in a fund compounded quarterly at 6%. For 30 years.

How much money in your Retirement account after years?

Formula

$$FV = PMT \left(\frac{(1+i)^n - 1}{i} \right)$$

FV = future value (final amount in the account)

PMT = periodic payment.

i = rate per compounding period. $= \frac{R}{m}$

n = total # of compounding periods.

$$FV = 2000 \cdot \left(\frac{\left(1 + \frac{0.06}{4} \right)^{120} - 1}{\frac{0.06}{4}} \right)$$
$$\approx 662576.38$$

How much interest have you earned for this account?

$(2000 \cdot 4) \cdot 30$ ← amount you put in

$662576.38 - 240000$

$= 422576.38$

Formula for interest: $FV - PMT \cdot n$

Sinking Funds:

Annuity: Given periodic payment → FV

Sinking fund: Given FV (target amount) →
how much should you pay periodically.

$$FV = PMT \left(\frac{(1+i)^n - 1}{i} \right)$$

$$PMT = \frac{FV}{\frac{(1+i)^n - 1}{i}} = FV \left(\frac{i}{(1+i)^n - 1} \right)$$

Note: For these formulas to work the periodic payment need to be made at the end of each compounding period.

E.g. Parents of newborn. Set up college fund.

Deposit yearly into the fund for 18 years

Rate: 7% compounded annually.

Goal. Have \$100,000 at the child's 18th birthday.

How much money should they contribute yearly?

$$PMT = 100000 \left(\frac{0.07}{(1+0.07)^{18} - 1} \right) = 2941.26$$

8 HW.

Bob : contribute \$2500 into a retirement account
every year for 26 years

6.4% compounded annually.

Then he stops.

leave the money in the account for the next

16 years compounded 6.4% annually

How much money in the account when Bob withdraws
it for retirement.

423 424.18

$$FV = 2500 \left(\frac{(1 + 0.064)^{26} - 1}{0.064} \right)$$

$$= 156931.31$$

$$A = (156931.31) (1 + 0.064)^{16}$$
$$= \boxed{423424.17}$$