

### 3.4. Present Value of an Annuity, Amortization

Goals ① Calculate the present value of an annuity

② Calculate the payment on a loan

③ Construct an amortization table

Present Value of an annuity

$$FV = PMT \left( \frac{(1+i)^n - 1}{i} \right)$$

↙ future value

We make periodic payments today, how much will the account be worth in the future.

Now, if we want to with-draw periodic amounts in the future, what amount should we deposit today?

E.g. A lending service offers 42-month loan at 6.6% compounded monthly.

You can afford payment of \$225/month.

How much can you borrow?

Formula

$$PV = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right)$$

Present value of the annuity

periodic payment

rate per period

total # of periods

$$PV = 225 \left( \frac{1 - \left( 1 + \frac{0.066}{12} \right)^{-42}}{\frac{0.066}{12}} \right)$$

In TI-84:

$$\frac{225 * \left( 1 - \left( 1 + (0.066/12) \right)^{-42} \right)}{\left( \frac{0.066}{12} \right)} \approx 8417.37$$

## A mortization Problem (loan payment)

An amortization of a debt is the process of paying it off in equal payments.

E.g. Bank loans you \$50,000 at 4.5% interest to buy a house. You agree to make monthly payments for next 15 years for a total of 180 payments. How much should the monthly payment be to pay off debt after 15 years.

$$PMT = PV \left( \frac{i}{1 - (1+i)^{-n}} \right)$$

$$PV = 50000$$

$$i = \frac{0.045}{12} = 0.0038; \quad (1.0038)^{-180} = 0.51$$

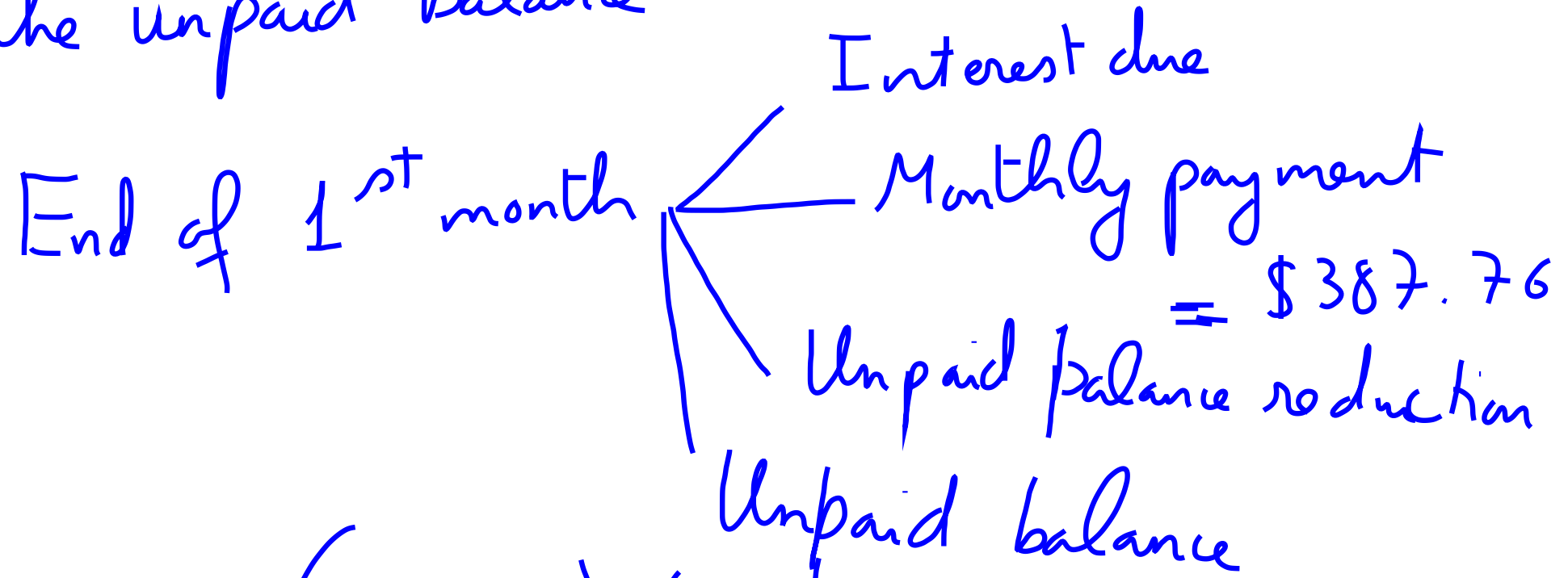
$$n = 180$$

$$50000 \left( \frac{0.0038}{1 - 0.51} \right) \approx 387.76$$

How much interest the bank earned?

$$(387.76) \cdot (180) - 50000 = \$19795.92$$

\* How much of each monthly payment is used for interest? and how much is used to reduce the unpaid balance?



$$\text{Interest} = (50000) \cdot (0.0038) = \$190$$

$$\text{Reduction} = \$387.76 - \$190 = \$197.76$$

$$\text{Unpaid Balance} = \$50000 - \$197.76 = 49802.24$$

End of second month

$$\text{Interest} = (49802.24) \cdot (0.0038) \\ = 189.24$$

$$\text{Reduction} = \$387.76 - \$189.24 =$$

$$\text{Unpaid balance} =$$