4.3. Gauss - Jordan Elimination

Goals: ① Reduced row echelon form of a matrix

② Gauss-Jordan Elimination.

Matrices associated with linear system of 3 variables.

E.g.

$$\begin{aligned} x + 2y - z &= 5 \\ 2x - 3y + z &= 3 \\ 3x - y - 2z &= 7 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & -3 & 1 & 3 \\ 3 & -1 & -2 & 7 \end{array}\right)$$

Possible final "nice" form

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$x = 3$
$y = 4$
$z = 5$

consistent system with unique solution.

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow$$ No solutions Inconsistent.

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow$$ Infinitely many solutions

Reduced Row Echelon form.

A matrix is in Reduced Row Echelon form if:

1. Each row consisting entirely of zeros is below any row having at least a nonzero entry
2. The leftmost nonzero element in each row is <u>1</u>.
3. All other elements in the column containing the left most 1 of a given row are zeros.
4. The left most 1 in any row is to the right of the left most 1 in the row above it

Gauss - Jordan Elimination

$$x + y - z = -2$$
$$2x - y + z = 5$$
$$-x + 2y + 2z = 1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right) \xleftarrow{R_2 + (-2)R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right) \xleftarrow{R_3 + R_1}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right) \xleftrightarrow{-\frac{1}{3}R_2} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & \boxed{1} & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right)$$

$$\xleftrightarrow{R_3 + (-3) R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right) \xrightarrow{\frac{1}{4}R_3} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & \boxed{-1} & -3 \\ 0 & 0 & 1 & 2 \end{array}\right) \xleftrightarrow{R_3 + R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{R_3 + R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right) \xleftrightarrow{R_1 - R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$x = 1$

$y = -1$

$z = 2$