

4.4. Basic operations on Matrices

Goals: ① Add and subtract Matrices.

② Multiply a number to a matrix.

③ Multiply matrices together.

① Add and subtract Matrices

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 1 \\ 8 & 7 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ 12 & 13 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 1 \\ 8 & 7 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -4 & -1 \end{pmatrix}$$

Add / Subtract Matrices . entry by entry.

Note: Dimensions have to be the same to add / subtract matrices

② Multiply a number to a matrix

$$13 \cdot \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 26 & 39 \\ 52 & 78 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 1 & \frac{3}{2} \\ 2 & 3 \end{pmatrix}$$

$$-\frac{1}{2} \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{3}{2} \\ -2 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -8 \\ 4 & -16 \end{pmatrix}$$

Do it entry by entry

③ Product of matrices

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + 3 \cdot 0 & 2 \cdot (-1) + 3 \cdot (-2) \\ 4 \cdot 1 + 6 \cdot 0 & 4 \cdot (-1) + 6 \cdot (-2) \end{pmatrix}$$

Ex. 9.

$$\begin{pmatrix} \boxed{5} & \boxed{7} \\ \boxed{3} & \boxed{8} \\ 2 & 6 \end{pmatrix} \begin{pmatrix} \boxed{4} \\ \boxed{-1} \end{pmatrix} = \begin{pmatrix} 5 \cdot 4 + 7 \cdot (-1) \\ 3 \cdot 4 + 8 \cdot (-1) \\ 2 \cdot 4 + 6 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 13 \\ 4 \\ 2 \end{pmatrix}$$

$$3 - by - 2 \quad 2 - by - 1$$



Product: $3 - by - 1$

In general, $(m \cdot by - n)(n - by - q) = (m - by - q)$

$$(3 - by - 4)(4 - by - 3) = 3 - by - 3$$

E.x

$$(1 \quad 2 \quad 3 \quad 4) \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} = 1 \cdot 5 + 2 \cdot 6 + 3 \cdot 7 + 4 \cdot 8 = 70$$

$\underbrace{\hspace{10em}}_{1 \cdot \text{by} \cdot 4} \quad \underbrace{\hspace{10em}}_{4 \cdot \text{by} \cdot 1}$

$$\begin{pmatrix} \boxed{3 \quad 1 \quad -1} \\ 2 \quad 0 \quad 3 \end{pmatrix} \begin{pmatrix} 1 \quad \boxed{6} \\ 3 \quad -5 \\ -2 \quad 4 \end{pmatrix} = \begin{pmatrix} 8 & 9 \\ -4 & 24 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{2 \cdot \text{by} \cdot 3} \quad \underbrace{\hspace{10em}}_{3 \cdot \text{by} \cdot 2}$

$\underbrace{\hspace{10em}}_{\text{product} : 2 \cdot \text{by} \cdot 2}$

Revenue of a car dealer.

Sell : 4 models : A, B, C, D.

In a given week: 10 cars model A
5 _____ B
8 _____ C
3 _____ D

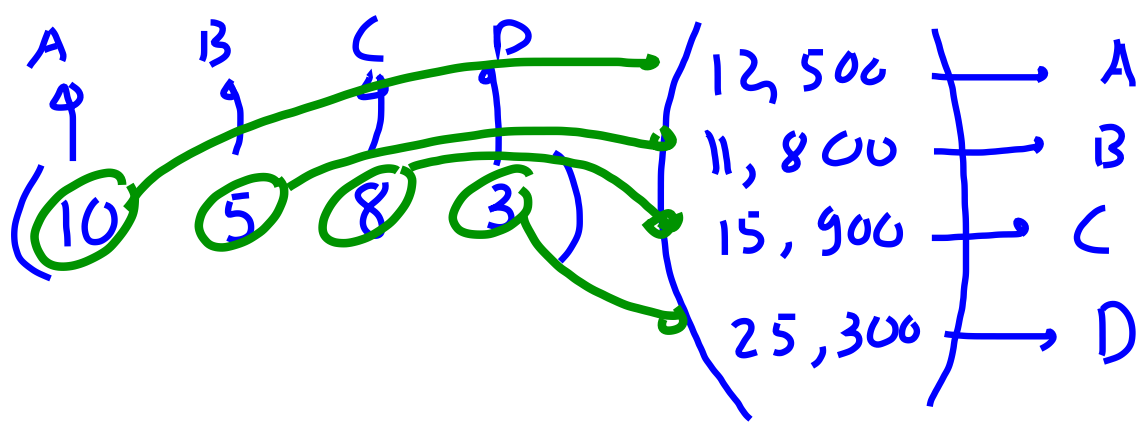
A car of model A sells for \$12,500

B \$11,800

C \$15,900

D \$25,300

} Revenue



$$= 10 \cdot (12,500) + 5 \cdot (11,800) + 8 \cdot (15,900) + 3 \cdot (25,300)$$

$$= 387100$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 6 & 15 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 4 & 15 \end{pmatrix}$$

Matrix Multiplication is NOT commutative.

$$AB \neq BA \text{ in general.}$$