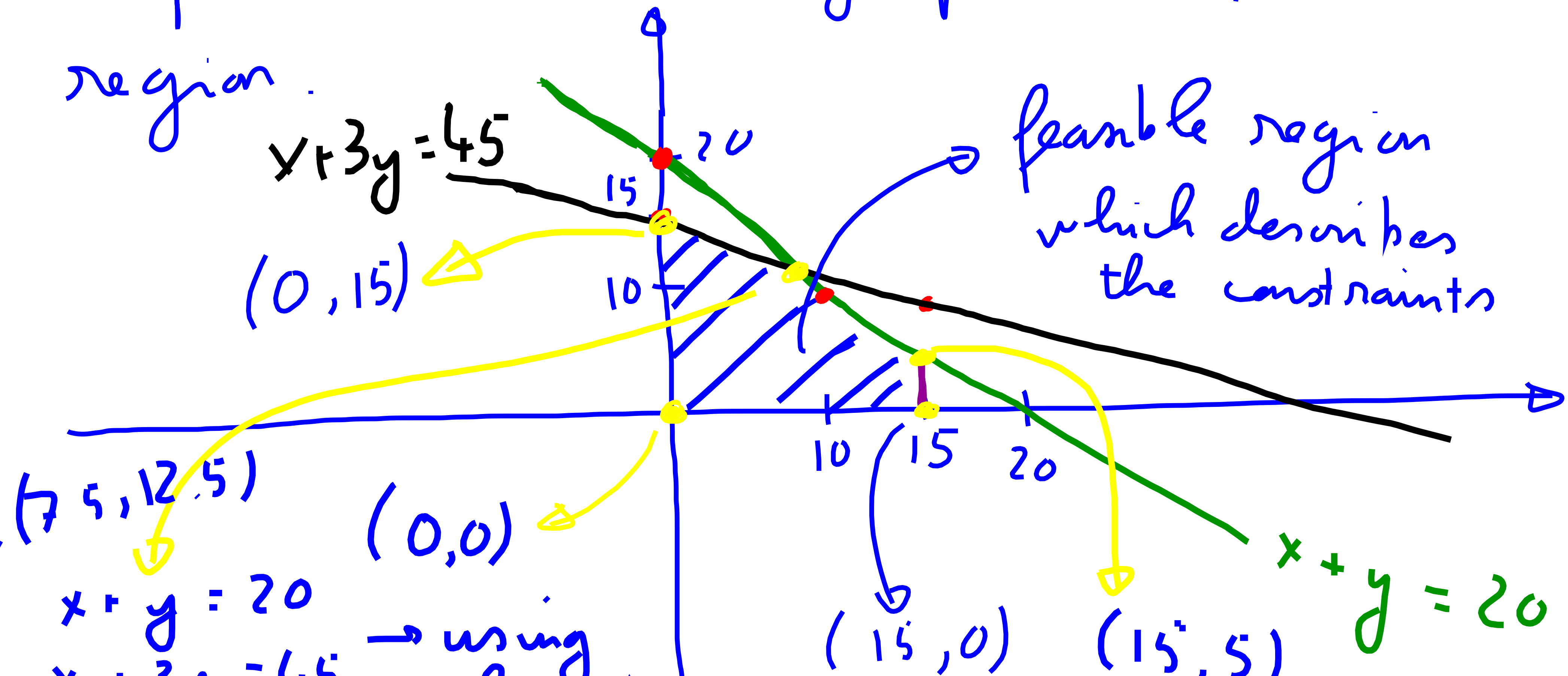
5.3. Linear Programming in 2 dimensions.

__Goal:__ Solve linear programming problems.

Ski manufacturer problem

In previous section, we graph the feasible region.



$x + y = 20$

$x + 3y = 45$ → using calculator

Profit for each trick ski : \$ 5

Profit for each slalom ski : \$10

Total profit = $5x + 10y$

$$z = 5x + 10y$$

<u>Goal</u>: Maximize the profit $z = 5x + 10y$.

① Find corner points of the feasible region.

(2) Plug corner points into profit function to determine the optimal solution.

Corner points	Profit $z = 5x + 10y$
$(0,0)$ →	0
$(15,0)$ →	75
$(15,5)$ →	125
$(0,15)$ →	150
$\boxed{(7.5, 12.5)}$ →	$\boxed{162.5}$

$x = 7.5$
$y = 12.5$
} Round down we can't exceed the bdry

optimal solution

$\boxed{\begin{array}{l} x = 7 \\ y = 12 \end{array}}$

Process of using the linear programming technique to find optimal solution.

1. Find the objective function.
   (In the example, it is the profit $z = 5x + 10y$)
2. Find the inequalities that describe the constraints
3. Graph the feasible region.
4. Find corner points of the feasible region.
5. Plug the corner points into the objective function to determine the optimal solution.

E.g. Boat manufacturer

2 types of boat
- 2-person boat
  - 0.9 hours for cutting
  - 0.8 hours for assembly
- 4-person boat
  - 1.8 hours for cutting
  - 1.2 hours for assembly

Maximum # of hours for cutting department is 864

———————— assembly ———— 672

Profit for each 2-person boat : \$25

———————— 4-person ———— : \$40

How many boats of each type to produce to maximize profit.

① Objective function:

$$z = 25x + 40y$$

② Constraints:

$$x \geq 0 \; ; \; y \geq 0$$

$$0.9x + 1.8y \leq 864$$

$$0.8x + 1.2y \leq 672$$

③ Graph feasible region

$$1.8y \leq 864 - 0.9x$$

$$y \leq \frac{864}{1.8} - \frac{0.9}{1.8}x$$

$$y \leq \frac{672}{1.2} - \frac{0.8}{1.2}x$$

Input to calculator

(4) Corner points | Objective function $z = 25x + 40y$

Corner points	Objective function $z = 25x + 40y$
(0,0)	0
(0,480)	19200
(840,0)	21000
(480,240)	21600 → max profit

$x = 480$

$y = 240$