7.2. Sets

Goals: (1) Understand set notation

(2) Perform set operation

(3) Solve applications.

Definition of a set: a set is a collection of objects.

Eg. $A = \{1, 2, 3\}$; $B = \{a, b, c, d, e, f\}$

$C = \{dog, cat, fish\}$

$D = \mathbb{R}$; $X = \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

(set of real numbers)

Notation: $e \in A$

This means e is an element of the set A

(e belongs to A, e is in A)

$A = \{1, 2, 3\}$ $1 \in A$; $2 \in A$

$X = \mathbb{Z}$ $-100 \in X$

$e \notin A$. This means that e is NOT an element of A

(e does not belong to A or e is not in A)

$X = \mathbb{Z}$ $2.5 \notin X$

$B = \{a, b, c, d, e, 1\}$. $g \notin B$

Empty set or null set.

Empty set is the set with no elements in it

Notation: $\{\}$ or ϕ

Eg Find all the real solutions to the equation

$$x^2 + 1 = 0$$

$$x^2 = -1$$

The set of real solution to this equation is the empty set.

Set Builder Notation:

$A = \{a, b, c, d, e, f, \ldots, z\}$ ⟶ listing the elements

Instead of listing the elements of the set, we can

set builder notation to describe sets.

$A = \{ x \mid x \text{ is a letter of the English alphabet} \}$

$B = \{ \ldots, -3, -2, -1, 0, 1, 2, 3 \ldots \}$

$B = \{ x \mid x \text{ is an integer} \}$

Subsets:

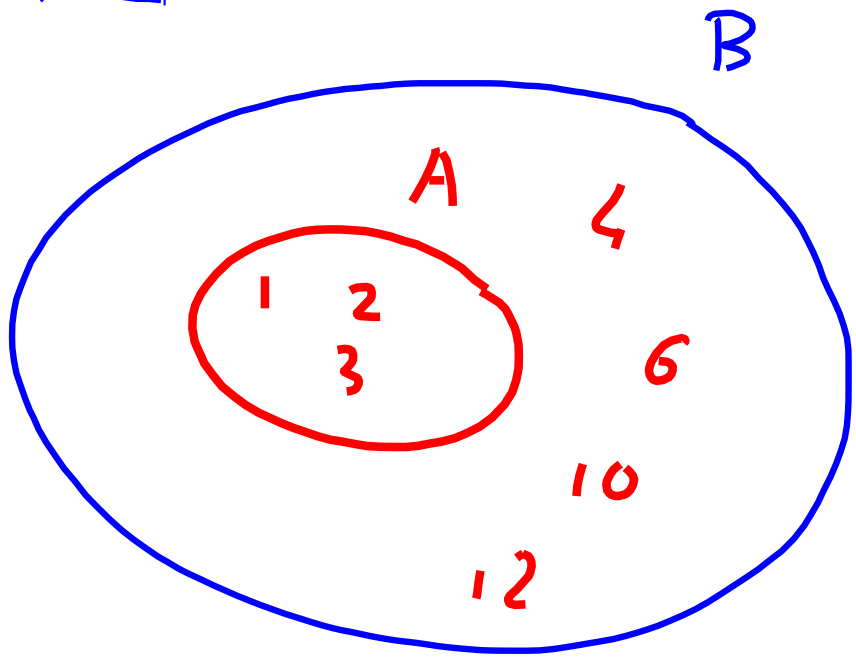
Definition: A and B are sets.

We say that A is a subset of B if every element of A is also an element of B

E.g. $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 6, 10, 12\}$

Notation for subset:

$A \subset B$



$X = \mathbb{Z}$, $Y = \mathbb{R}$

$X \subset Y$; $Y \not\subset X$

Note: ϕ is a subset of every set.

Eg. $A = \{5, 7, 9\}$

Find all the possible subsets of A.

ϕ, $\{5\}$, $\{7\}$, $\{9\}$

$\{5, 7\}$; $\{5, 9\}$; $\{7, 9\}$

$\{5, 7, 9\}$

Set operations

(1) Union of sets

The union of 2 sets A and B is the set

whose elements are the elements of A and elements of B

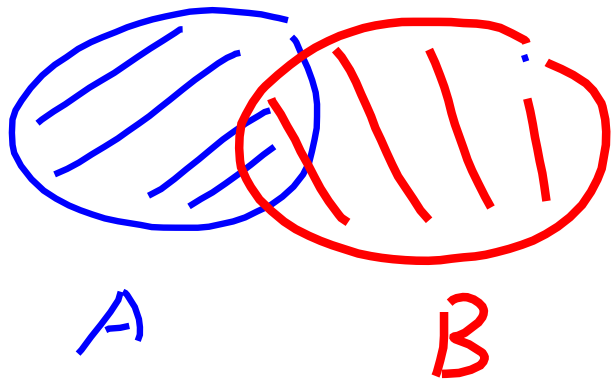
Notation: $A \cup B$

$A = \{1, 2, 3\}$ $B = \{a, b\}$

$A \cup B = \{1, 2, 3, a, b\}$

$X = \{2, 4\}$, $Y = \{2, 5\}$

$X \cup Y = \{2, 4, 5\}$



Intersection of sets

The intersection of 2 sets A and B is the set of all elements that are common to both A and B.

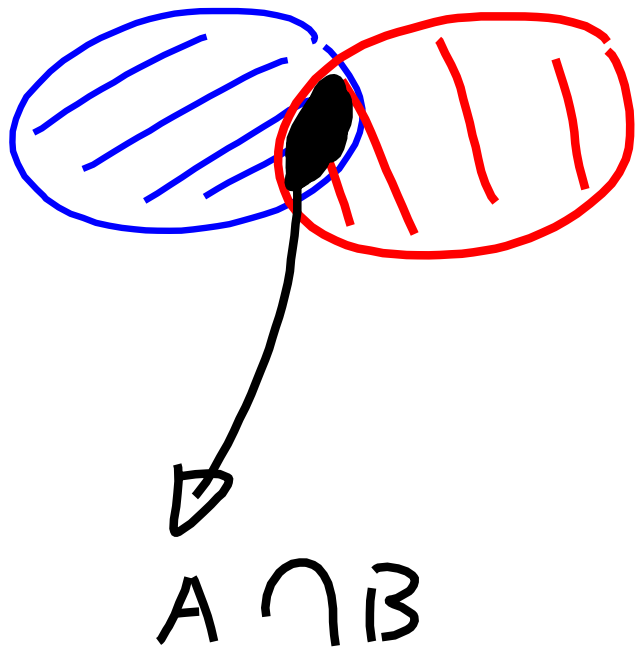
Notation: $A \cap B$

$A = \{2, 4\}$, $B = \{2, 5\}$

$A \cap B = \{2\}$

$A = \{1, 2, 3\}$, $B = \mathbb{Z}$

$A \cap B = \{1, 2, 3\}$

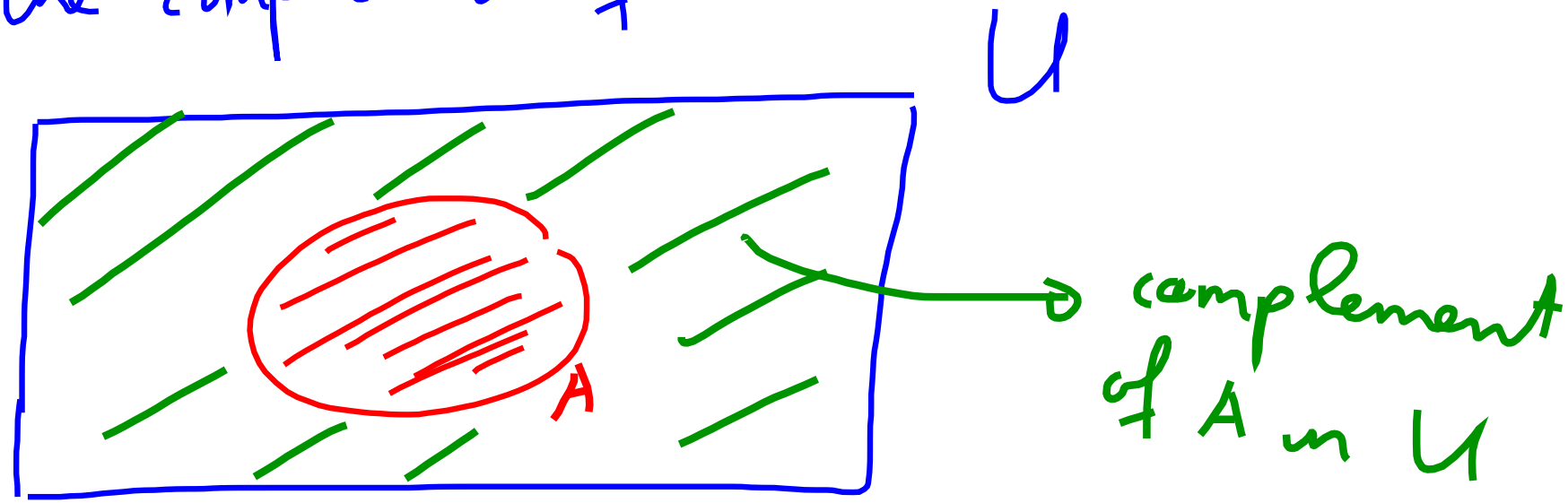


$A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$A \cap B = \emptyset$

If 2 sets have no elements in common, they are called <u>disjoint</u>.

(3) The complement of a set



If A is a subset of U, the complement of A in U is the set of elements that are in U but not in A.

Notation: A' : complement of A in U

$$A' = \{ x \mid x \in U \text{ and } x \notin A \}.$$

E.g. $U = \{1, 2, 3, \ldots, 10\}$; $A = \{1, 3, 5, 7, 9\}$

$$A' = \{2, 4, 6, 8, 10\}$$

Notation: $n(A)$ = # of elements in A.

In this case $n(A) = 5$

Application.

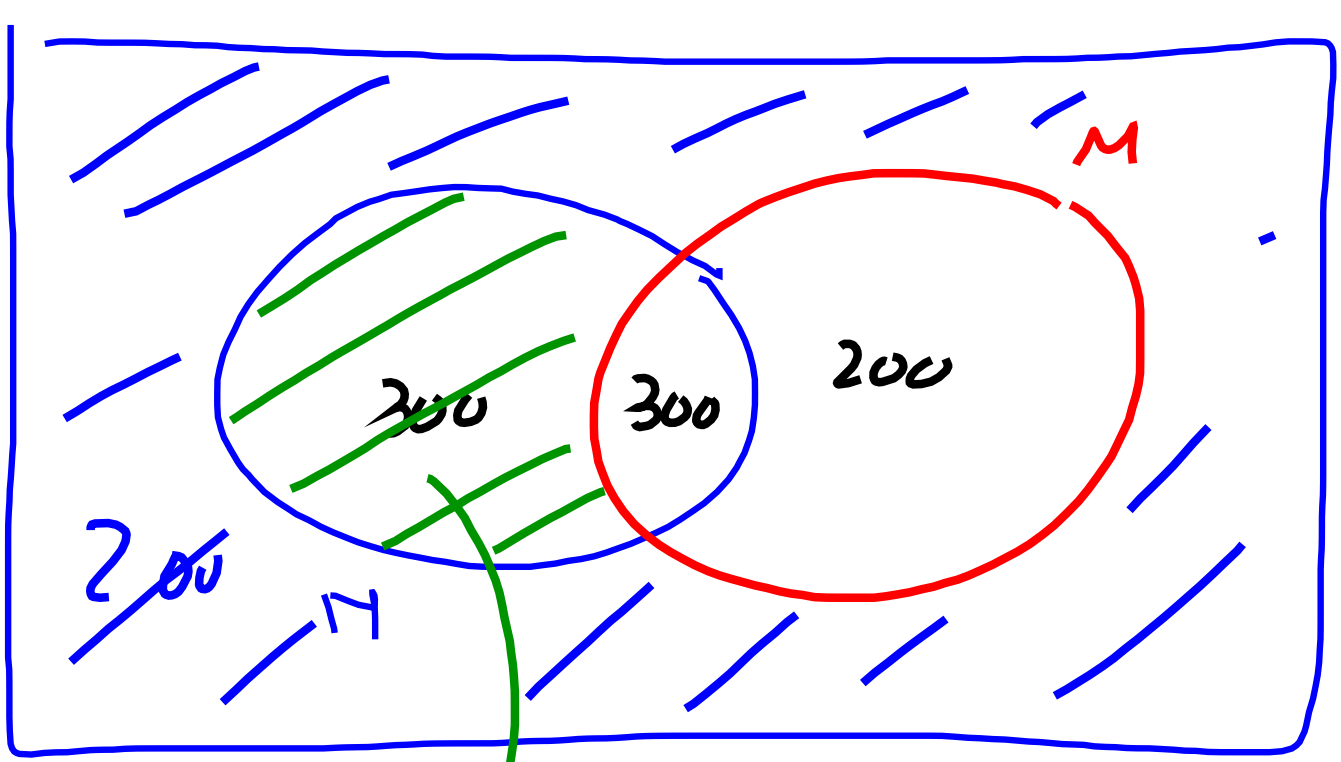
Survey of 1000 commuters.

600 listen to the news $\longrightarrow N$

500 listen to music $\longrightarrow M$

300 listen to both.

Question: Find $n(N \cap M')$



$N \cap M'$

$n(N \cap M') = 300$