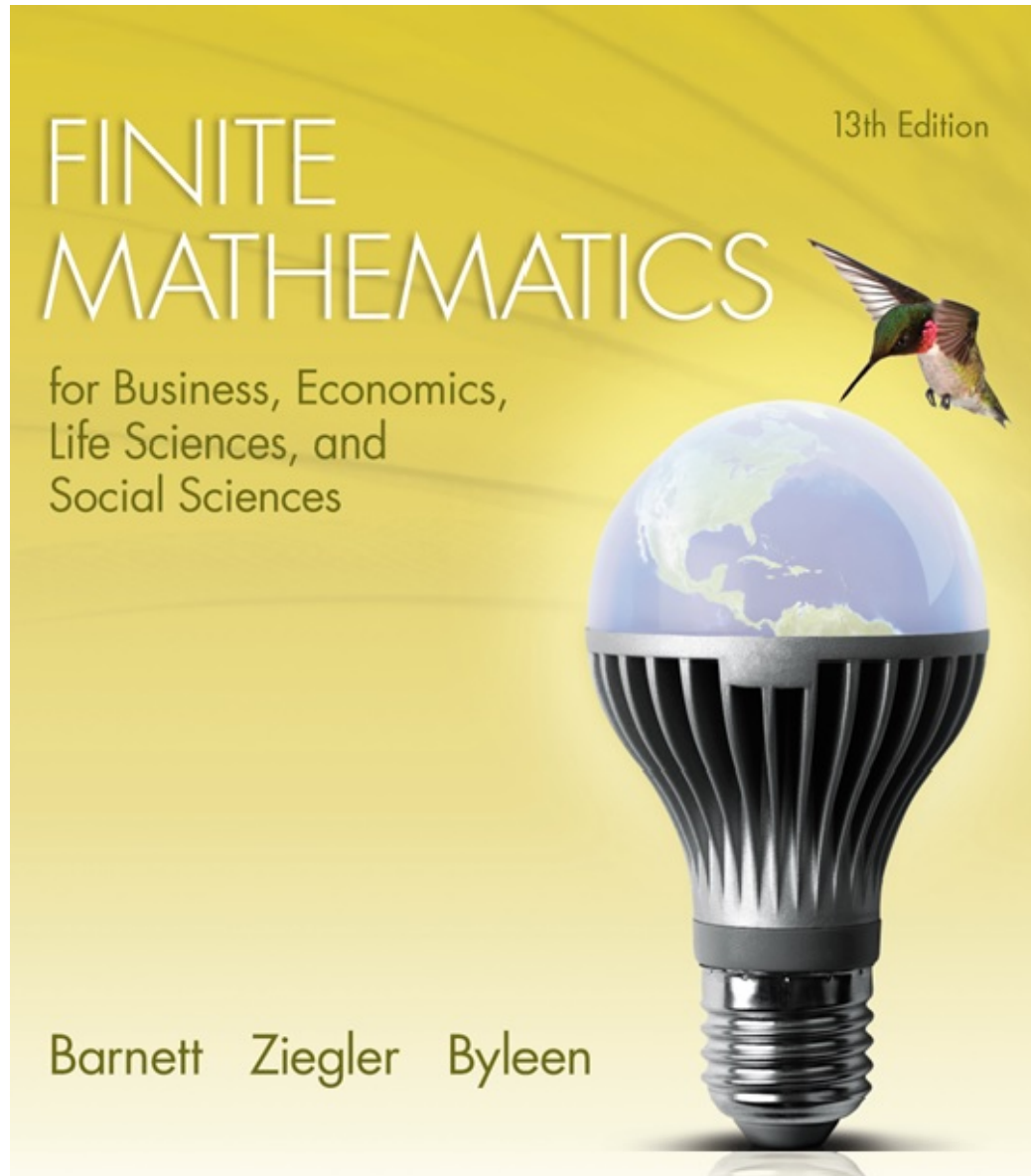




Chapter 7

Logic, Sets, and Counting

Section 2 Sets

Learning Objectives for Section 7.2

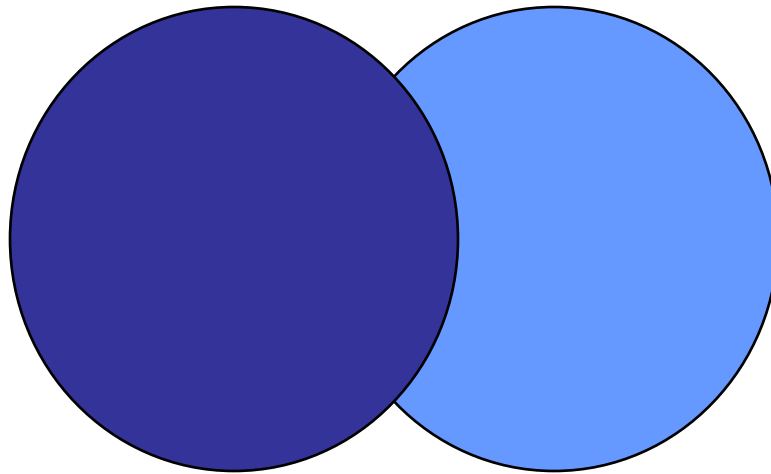
Sets



- The student will be able to identify and use set properties and set notation.
- The student will be able to perform set operations.
- The student will be able to solve applications involving sets.

7.2 Sets

This section will discuss the symbolism and concepts of set theory.



Set Properties and Set Notation



- **Definition:** A **set** is any collection of objects specified in such a way that we can determine whether a given object is or is not in the collection.

- **Notation:**

$$e \in A$$

means “ e is an element of A ”, or “ e belongs to set A ”.

The notation

$$e \notin A$$

means “ e is not an element of A ”.

Set Properties and Set Notation (continued)



- **Example.** A is the set of all the letters in the alphabet.

We write that as

$$A = \{ a, b, c, d, e, \dots, z \}.$$

We use capital letters to represent sets. We list the elements of the set within braces. The three dots ... indicate that the pattern continues. This is called the **roster method** of specifying the set.

- A is a set because we can determine whether an object is or is not in the collection. For example, $3 \notin A$.

Null Set



- **Example.** What are the real number solutions of the equation

$$x^2 + 1 = 0?$$

Null Set



- **Example.** What are the real number solutions of the equation

$$x^2 + 1 = 0?$$

- **Solution:** There are no real number solutions of this equation since no real number squared added to one can ever equal 0. We represent the solution as the null set, written $\{ \}$ or $\emptyset$.

Set Builder Notation

- Sometimes it is convenient to represent sets using **set builder notation**. For example, instead of representing the set A (letters in the alphabet) by the roster method, we can use

$$A = \{x \mid x \text{ is a letter of the English alphabet}\}$$

which means the same as $A = \{a, b, c, d, e, \dots, z\}$

- **Example.**

$$\{x \mid x^2 = 9\} = \{3, -3\}$$

This is read as “the set of all x such that the square of x equals 9.” This set consists of the two numbers 3 and -3.

Subsets



A is a **subset** of B if every element of A is also contained in B . This is written

$$A \subset B.$$

For example, the set of integers

$$\{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$$

is a subset of the set of real numbers.

Formal Definition:

$A \subset B$ means “if $x \in A$, then $x \in B$.”

Subsets (continued)

Claim: $\emptyset$ (the null set) is a subset of every set.

To verify this statement, pick an arbitrary set A , and apply the definition of subset:

If $x \in \emptyset$, then $x \in A$.

Since the null set contains no elements, the statement $x \in \emptyset$

is false. We know that $p \rightarrow q$ is true if p is false. Since p is false, we conclude that the conditional statement is true. Therefore, the null set is a subset of every set.

Number of Subsets



Example: List all the subsets of set $A = \{\text{bird, cat, dog}\}$ For convenience, we will use the notation $A = \{b, c, d\}$ to represent set A .

Number of Subsets



Example: List all the subsets of set $A = \{\text{bird, cat, dog}\}$
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Solution: $\emptyset$ is a subset of A . We also know that every set is a subset of itself, so $A = \{b, c, d\}$ is a subset of set A .

What two-element subsets are there? $\{b, c\}$, $\{b, d\}$, $\{c, d\}$

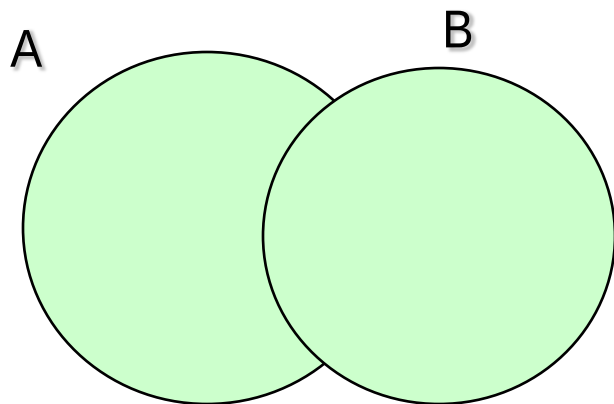
What one-element subsets? $\{b\}$, $\{c\}$ and $\{d\}$.

There are a total of 8 subsets of set A .

Union of Sets

The **union** of two sets A and B is the set of all elements formed by combining all the elements of set A and all the elements of set B into one set. It is written $A \cup B$.

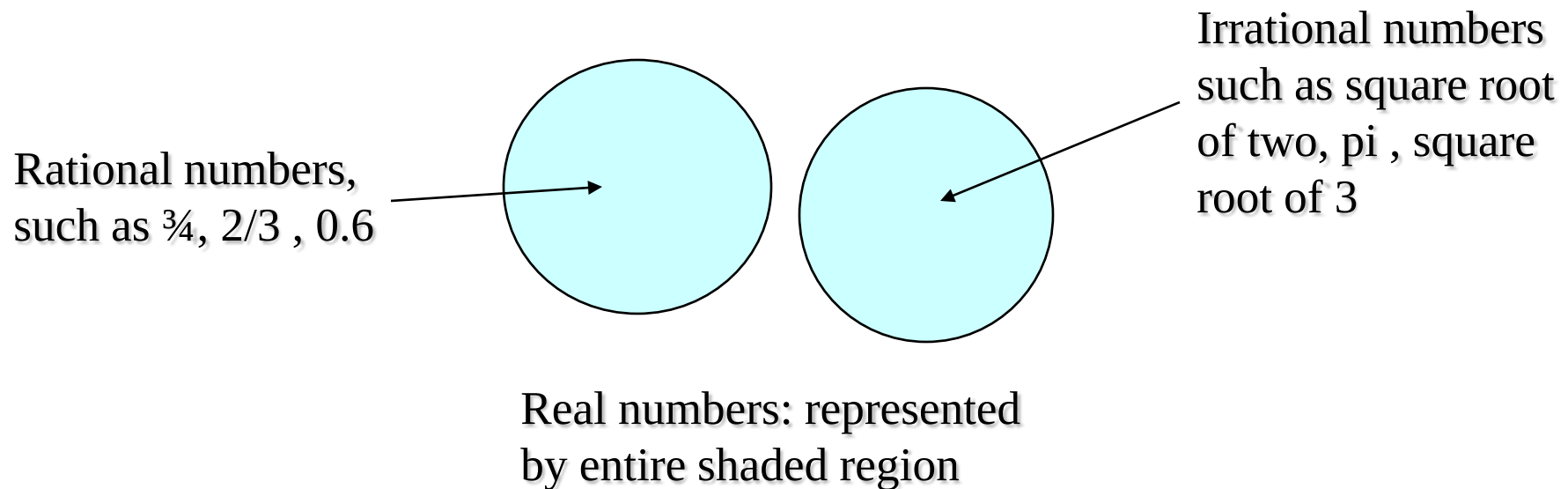
$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}.$$



In the Venn diagram on the left, the union of A and B is the entire region shaded.

Example of Union

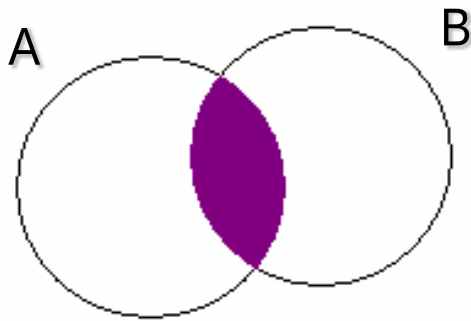
The union of the rational numbers with the set of irrational numbers is the set of real numbers. Rational numbers are those numbers that can be expressed as fractions, while irrational numbers are numbers that cannot be represented exactly as fractions, such as $\sqrt{2}$.



Intersection of Sets

The **intersection** of two sets A and B is the set of all elements that are common to both A and B . It is written $A \cap B$.

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}.$$



In the Venn diagram on the left, the intersection of A and B is the shaded region.

Disjoint Sets



If two sets have no elements in common, they are said to be **disjoint**. Two sets A and B are disjoint if

$$A \cap B = \emptyset.$$

Example: The rational and irrational numbers are disjoint.

The Universal Set



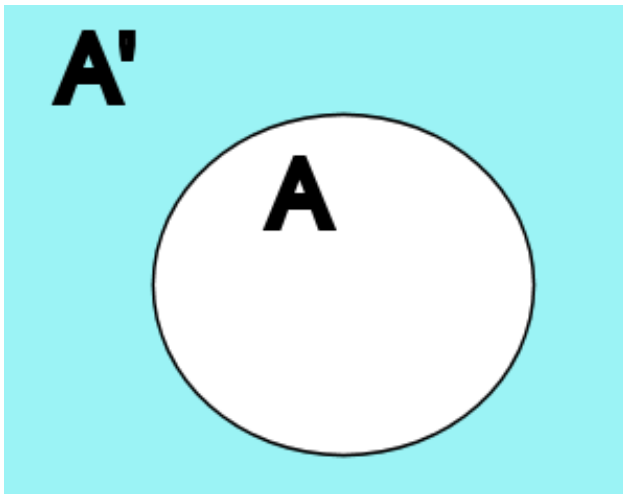
The set of all elements under consideration is called the **universal set** U .

For example, when discussing numbers, the universal set may consist of the set of real numbers. All other types of numbers (integers, rational numbers, irrational numbers) are subsets of the universal set of real numbers.

The Complement of a Set

The **complement** of a set A is defined as the set of elements that are contained in U , the universal set, but not contained in set A . The symbolism and notation for the complement of set A are

$$A' = \{x \in U \mid x \notin A\}$$



In the Venn diagram on the left, the rectangle represents the universe. A' is the shaded area outside the set A .

Application

A marketing survey of 1,000 commuters found that 600 answered listen to the news, 500 listen to music, and 300 listen to both. Let N = set of commuters in the sample who listen to news and M = set of commuters in the sample who listen to music. Find the number of commuters in the set $N \cap M$

The number of elements in a set A is denoted by $n(A)$, so in this case we are looking for

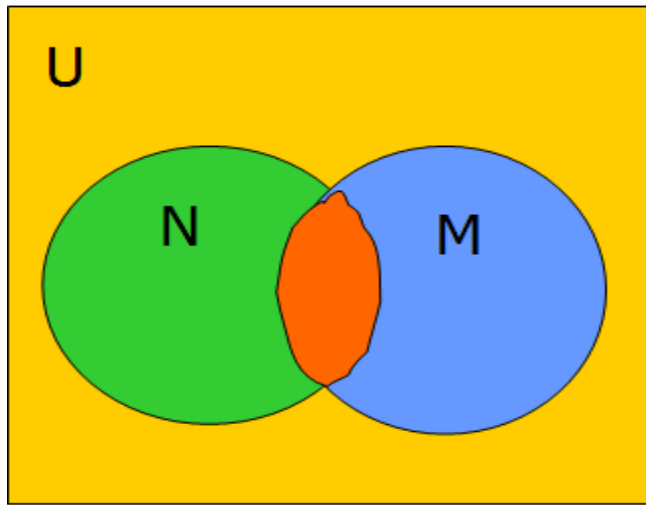
$$n(N \cap M)$$

Solution

The study is based on 1000 commuters, so $n(U)=1000$.

The number of elements in the four sections in the Venn diagram need to add up to 1000.

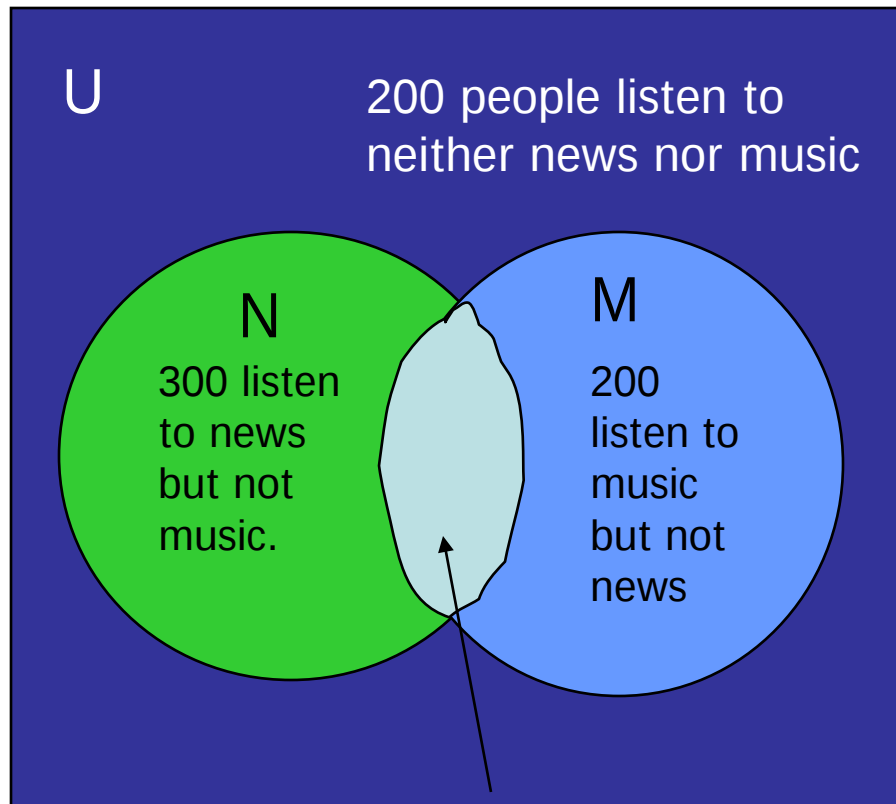
The red part represents the commuters who listen to both news and music. It has 300 elements.



The set N (news listeners) consists of a green part and a red part. N has 600 elements, the red part has 300, so the green part must also be 300.

Continue in this fashion.

Solution (continued)



$$N \cap M'$$

is the green part,
which contains
300 commuters.