Section 7.4. Permutations and Combinations

Goals:

1. Understand and Compute Factorials
2. Calculate permutations
3. Calculate Combinations
4. Solve application problems.

1. Factorials: are used to calculate permutations and combinations

$n!$ means the product of the first n whole numbers

$1! = 1$; $2! = 2 \cdot 1 = 2$; $3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$; $5! = 5 \cdot \underbrace{4 \cdot 3 \cdot 2 \cdot 1}_{} = 120$

$5 \cdot (4!)$

$n! = n \cdot (n-1)(n-2) \cdot (n-3) \cdots 2 \cdot 1$

$n! = n \cdot ((n-1)!)$

Note: $0! = 1 \longrightarrow$ by default.

E.g. Calculate $\frac{16!}{15!} = \frac{16 \cdot \cancel{15} \cdot \cancel{14} \cdot \cancel{13} \cdots \cancel{2} \cdot \cancel{1}}{\cancel{15} \cdot \cancel{14} \cdot \cancel{13} \cdot \cancel{12} \cdots \cancel{2} \cdot \cancel{1}} = 16$

$$\frac{9!}{7!} = \frac{9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdots \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdots \cancel{1}} = 72$$

$$\frac{100!}{(98!)(2!)} = \frac{100 \cdot 99 \cdot \cancel{98} \cdot \cancel{97} \cdots \cancel{1}}{(\cancel{98} \cdot \cancel{97} \cdots \cancel{1}) \cdot (2)} = \frac{100 \cdot 99}{2}$$

$$= 4950$$

② Permutation

E.g. Family has 4 kids: Ann, Jen, Billy, Tom

Argue who sits where in the family car which has 4 passenger seats. How many possible seating arrangements are there?

$$\boxed{A}\boxed{J}\boxed{B}\boxed{T} \neq \boxed{J}\boxed{B}\boxed{A}\boxed{T} \neq \boxed{T}\boxed{B}\boxed{A}\boxed{J}$$

How many choices for the first seat? 4

2nd seat? 3

3rd — ? 2

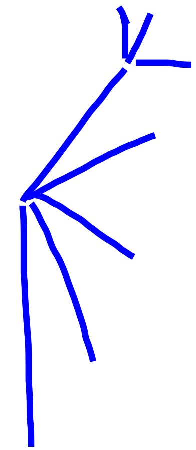
4th seat? 1

By mult. principle: $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$

E.g. 16 pool balls

Choose 3 of them, arrange them in line

(the order matters)



8	16	1

$\neq$

1	8	16

1st position: 16 choices

2nd position: 15 choices

3rd position: 14 choices

By mult. principle

$16 \cdot 15 \cdot 14 = 3360$

Note: $16 \cdot 15 \cdot 14 = \dfrac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdots 1}{13 \cdot 12 \cdots 1}$

$= \dfrac{16!}{13!} = \dfrac{16!}{(16-3)!}$

Permutation $P(n,r) = \dfrac{n!}{(n-r)!}$

$P(n,r)$ means the # of ways to choose r objects from n different objects where the order matters.

E.g. Choose 3 balls from 16 balls, order matters.

So answer $P(16,3) = \dfrac{16!}{(16-3)!} = \dfrac{16!}{13!} = 16 \cdot 15 \cdot 14$

Choose 4 people from 4 people: $P(4,4)$

$$P(4,4) = \frac{4!}{(4-4)!} = \frac{4!}{0!}$$

$$= 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Ex. 5 people (Ann, Jen, Billy, Tom, Ben)

Choose 1 president, 1 V.P.

How many ways can you do this?

$$P(5,2) = \frac{5!}{3!} = 5 \cdot 4 = 20$$

(3) Combinations.

Order doesn't matter.

Previous example: choose 2 people to invite to dinner

$\{Ann, Jen\} = \{Jen, Ann\}$

Permutation:	Combination
{Ann, Billy}	Ann, Billy
{Billy, Ann}	
{Jen, Ben}	Jen, Ben
{Ben, Jen}	

So, permutation will have twice as many choices as combination

Ans for combination: $\frac{20}{2} = 10$

$$\frac{P(5,2)}{2!} = \frac{5!}{2!\,3!}$$

Combination $C(n,r) = \dfrac{n!}{r!\,(n-r)!}$

$C(n,r)$ = # of ways to choose r objects from n different objects where order doesn't matter.

Eg. State lottery: select a set of 6 numbers from 49 numbers. To win the lottery, you must select the correct set of 6 numbers.

How many tickets should be printed.

$$C(49,6) = \frac{49!}{6!\,43!} = \frac{49\cdot 48\cdot 47\cdot 46\cdot 45\cdot 44}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

E.g.

(1) Find the # of different poker hands.

(Standard 52 card deck)

$$C(52,5) = \frac{52!}{5!\,47!} = \boxed{2,598,960}$$

(2) # of ways to get exactly one pair

13 kinds: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

4 suits in a kind: spades, diamonds, hearts, clubs

$$C(13,1)\cdot C(4,2)\cdot C(12,3)\cdot C(4,1)\cdot C(4,1)\cdot C(4,1)$$

$$\frac{13!}{12!}\cdot\frac{4!}{2!\,2!}\cdot\frac{12!}{3!\,9!}\cdot 4\cdot 4\cdot 4 = \boxed{1,098,240}$$

$13\cdot 6\cdot$

② # of ways to get a full house?

3 of a kinds, and 2 of a kinds

$$C(13,1) \cdot C(4,2) \cdot C(12,1) \cdot C(4,3)$$