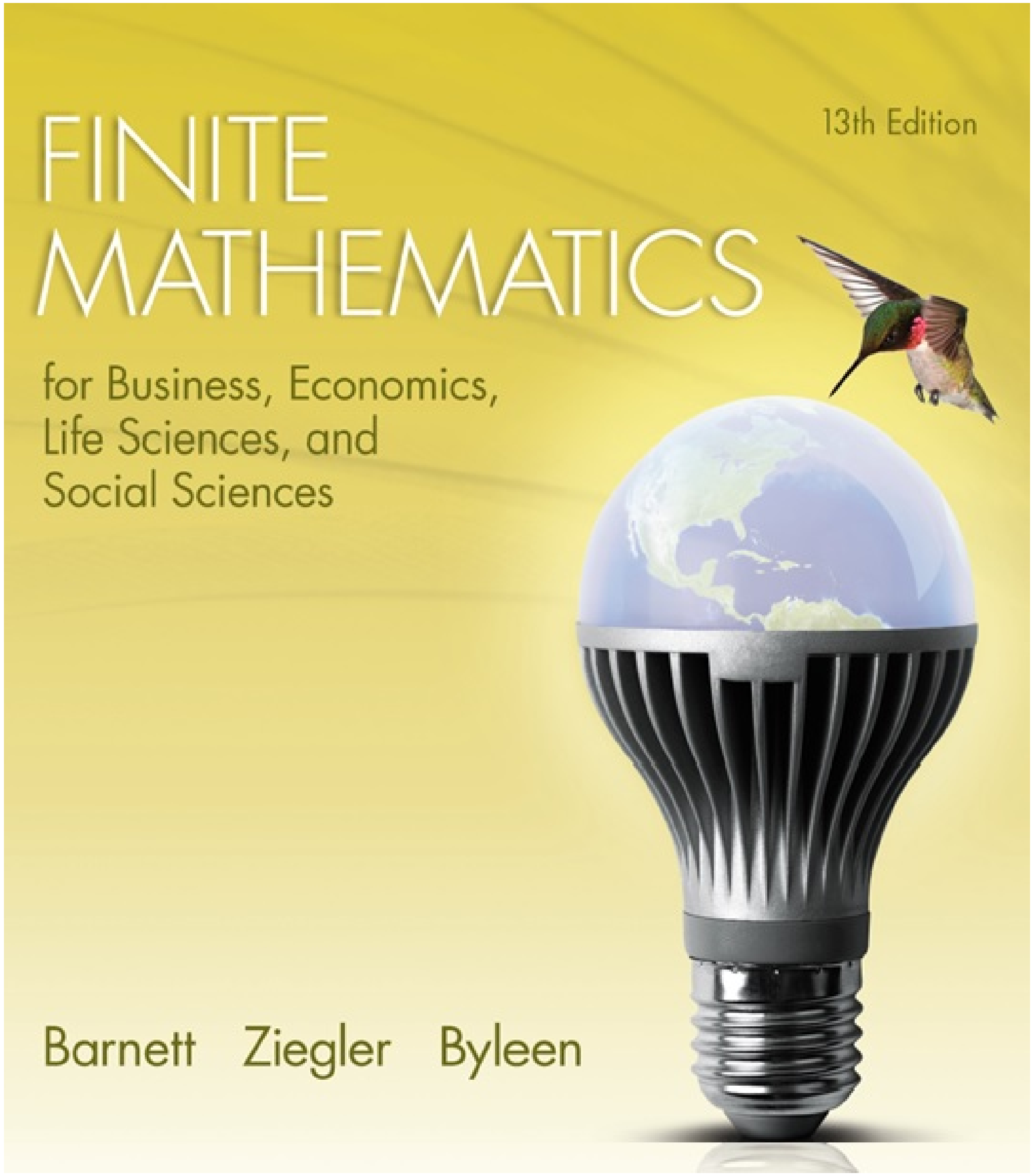

Chapter 7

Logic, Sets, and Counting

Section R
Review

Chapter 7 Review
Important Terms, Symbols, Concepts

- 7.1 Logic
  - A **proposition** is a statement (not a question or command) that is either true or false.
  - If p and q are propositions, then the **compound propositions**

    $$\neg p,\ p \vee q,\ p \wedge q,\ p \to q$$

    can be formed using the negation symbol $\neg$ and the connectives $\vee$, $\wedge$, $\to$.
  - These propositions are called **not *p***, ***p* or *q***, ***p* and *q***, and **if *p* then *q***, respectively (or **negation**, **disjunction**, **conjunction**, and **conditional**, respectively.)

Chapter 7 Review

- 7.1 Logic (continued)
  - With any conditional proposition $p \rightarrow q$ we associate the proposition $q \rightarrow p$, called the **converse** of $p \rightarrow q$, and the proposition $\neg q \rightarrow \neg p$, called the **contrapositive** of $p \rightarrow q$.
  - A **truth table** for a compound proposition specifies whether it is true or false, for any assignment of truth values to its variables.
  - A proposition is a **tautology** if each entry in its column of the truth table is T, a **contradiction** if each entry is F, and a **contingency** if at least one entry is T and at least one entry is F.

Chapter 7 Review

- 7.1 Logic (continued)
  Consider the rows of the truth tables for the compound propositions P and Q.
  - If whenever P is true, Q is also true, we say that P **logically implies** Q, and write $P \Rightarrow Q$.
  - We call $P \Rightarrow Q$ a **logical implication**.
  - If the compound propositions P and Q have identical truth tables we say that P and Q are **logically equivalent**, and write $P \equiv Q$.
  - We call $P \equiv Q$ a **logical equivalence**.

Chapter 7 Review

- 7.1 Logic (continued) A truth table will establish that **any conditional proposition is logically equivalent to its contrapositive**.
- 7.2 Sets
  - A **set** is a collection of objects specified in such a way that we can tell whether any given object is or is not in the collection.
  - Each object in a set is called a **member** or **element** of the set. If a is an element of the set A, we write $a \in A$.

Chapter 7 Review

- 7.2 Sets (continued)
  - A set without any elements is called the **empty** or **null set**, denoted Ø.
  - A set can be described by listing its elements, or by giving a rule that determines the elements of the set. If $P(x)$ is a statement about x, then $\{x \mid P(x)\}$ denotes the set of all x such that $P(x)$ is true.
  - A set is **finite** if its elements can be counted and there is an end; a set such as the positive integers, in which there is no end in counting its elements, is **infinite.**

Chapter 7 Review

- 7.2 Sets (continued)
  - We write $A \subset B$ and say that A is a **subset** of B if each element of A is an element of B. We write $A = B$ and say that sets A and B are **equal** if they have exactly the same elements. The empty set Ø is subset of every set.
  - If A and B are sets, then

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

is called the **union** of A and B, and

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

is called the **intersection** of A and B.

Chapter 7 Review

- 7.2 Sets (continued)
  - **Venn diagrams** are useful in visualizing set relationships.
  - If $A \cap B = \varnothing$, the sets A and B are said to **disjoint**.
  - The set of all elements under consideration in a given discussion is called the **universal set** U. The set

$$A' = \{x \in U \mid x \notin A\}$$

is called the **complement of A** (relative to U).

Chapter 7 Review

- 7.3 Basic Counting Principles
  - If *A* and *B* are sets, then the number of elements in the union of *A* and *B* is given by the **addition principle** for counting.
  - If the elements of a set are determined by a sequence of operations, tree diagrams can be used to list all combined outcomes. To count the number of combined outcomes without using a tree diagram, we use the **multiplication principle** for counting.

Chapter 7 Review

- 7.4 Permutations and Combinations
  - The product of the first n natural numbers, denoted $n!$, is called ***n* factorial**:

    $$n! = n(n-1)(n-2)\ldots.(2)(1)$$

    $$0! = 1$$

    $$n! = n(n-1)!$$

  - A **permutation** of a set of distinct objects is an arrangement of the objects in a specific order without repetition. The number of permutations of a set of n distinct objects is given by $P_{n,n} = n!$

Chapter 7 Review

- 7.4 Permutations and Combinations (continued)
  - A **permutation** of a set of *n* distinct objects taken *r* at a time without repetition is an arrangement of *r* of the *n* objects in a specific order. The order of the objects matters.
  - The number of permutations of *n* distinct objects taken *r* at a time without repetition is given by

$$P_{n,r} = \frac{n!}{(n-r)!} \qquad 0 \le r \le n$$

Chapter 7 Review

- 7.4. Permutations and Combinations (continued)
  - A **combination** of a set of *n* distinct objects taken *r* at a time without repetition is an *r*-element subset of the set of *n* objects. The order of the objects does not matter.
  - The number of combinations of *n* distinct objects taken *r* at a time without repetition is given by

$$C_{n,r} = \frac{P_{n,r}}{r!} = \frac{n!}{r!(n-r)!} \qquad 0 \le r \le n$$