

8.1. Sample Spaces, Events and Probability.

Goals: ① Understand the concepts of an experiment, outcomes, sample space and event.

② Calculate probability of an event.

A random experiment is a process which produces a number of possible outcomes. Outcomes cannot be predicted with absolute certainty.

E.g ① Toss a fair coin $\rightarrow$ experiment

Possible outcomes: H or T

② Toss 2 fair coins $\rightarrow$ experiment

possible outcomes: HH, HT, TH, TT

③ Roll a die $\leftarrow$ experiment
possible outcomes 1, 2, 3, 4, 5, 6

Sample Space: of an experiment is the set whose elements are the possible outcomes of the experiment.

E.g. Toss a coin $S = \{H, T\}$

Toss 2 coins $S = \{HH, TT, TH, HT\}$

Event: An event is a subset of the sample space

E.g. Toss 2 coins

E is the event where we have at least 1 head

$$E = \{HH, TH, HT\}$$

Roll a die

$\bar{E}$: we get an even #

$$\bar{E} = \{2, 4, 6\}$$

E.g. Experiment. Pick a card from a standard deck.

$$\text{Sample space} = S = \left\{ \begin{array}{l} 13 \text{ clubs, } 13 \text{ spades, } 13 \\ \text{hearts, } 13 \text{ diamonds} \end{array} \right\}$$

→ 52 elements in S

Event E the card is red. E has 26 elements

The probability of an event E , denoted $P(E)$

is defined as

$$P(E) = \frac{n(E)}{n(S)} = \frac{\# \text{ of elements in } E}{\# \text{ of elements in } S}$$

Ex. Roll 2 dice. $(2,2); (1,2); (2,1)$

① $S = ?$

② E sum is equal 7. $E = ?$

③ $P(E) = ?$

① S has 36 elements

② E has 6 elements

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Ex. Experiment: pick a card from a 52 card deck.

Event E: get an ace or a king

$$P(E) = \frac{8}{52} = \frac{2}{13}$$

Event F: get a red king $P(F) = \frac{2}{52} = \frac{1}{26}$

Event H: neither a spade nor a jack

$$P(H) = \frac{36}{52} = \frac{9}{13}$$