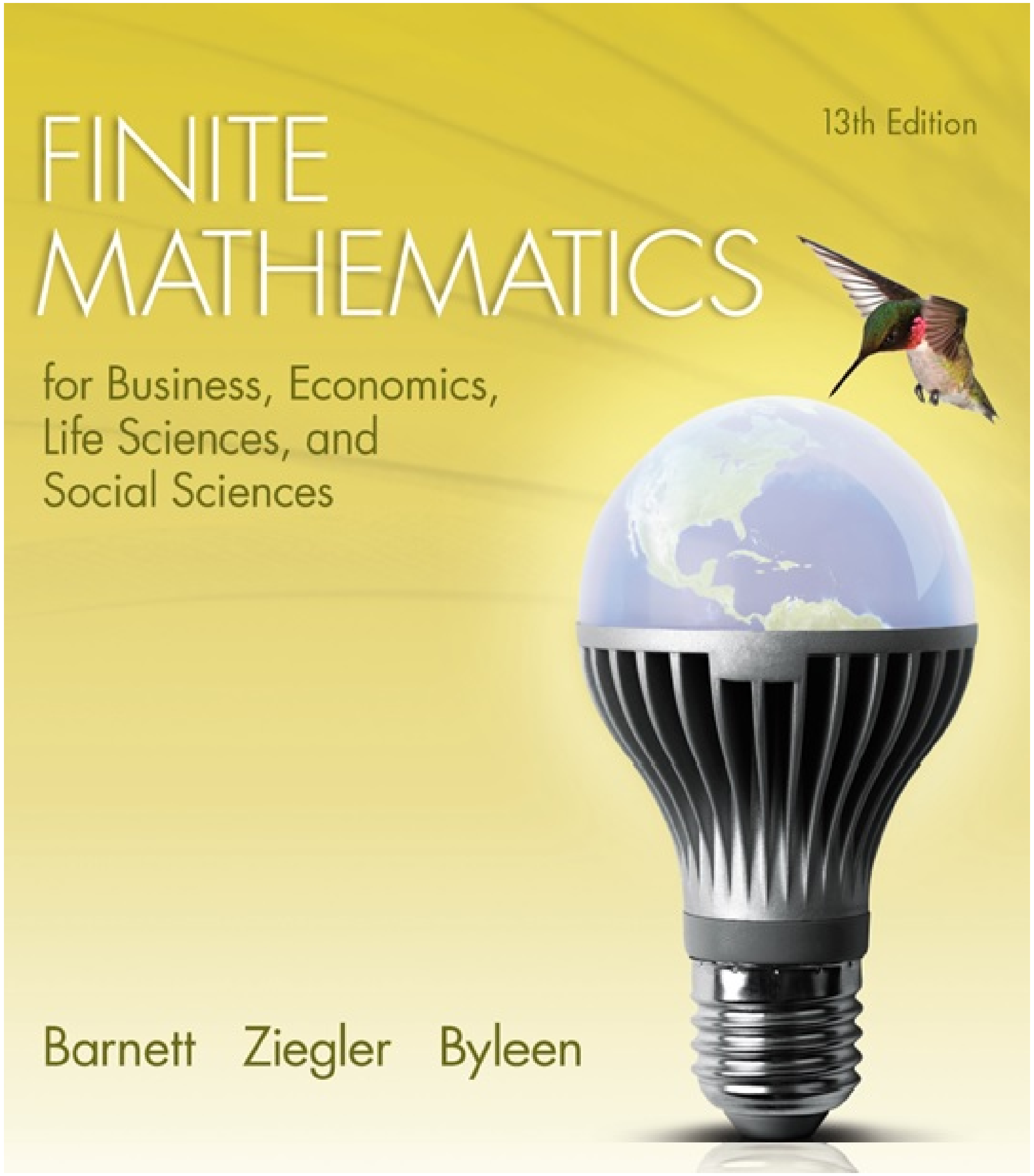


Chapter 8

Probability

Section 1

Sample Spaces, Events, and Probability

Learning Objectives for Section 8.1 Probability

- The student will be able to define and identify what is meant by an experiment.
- The student will be able to construct sample spaces and identify events.
- The student will be able to calculate probabilities of simple events.
- The student will be able to apply the equally likely assumption of probability.

Sample Space, Events, Probability

In this chapter, we will study the topic of probability which is used in many different areas including insurance, science, marketing, government and many other areas.

Blaise Pascal
1623-1662

http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pascal.html

- Blaise Pascal was born on 19 June 1623 in Clermont (now Clermont-Ferrand), Auvergne, France. He died 19 Aug 1662 in Paris, France.
- In correspondence with Fermat he laid the foundation for the theory of probability. This correspondence consisted of five letters and occurred in the summer of 1654. They considered the dice problem, already studied by Cardan, and the problem of points also considered by Cardan and, around the same time, Pacioli and Tartaglia.

Blaise Pascal (continued)

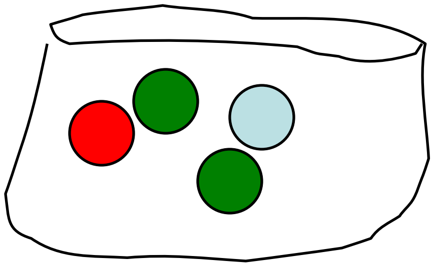
The dice problem asks how many times one must throw a pair of dice before one expects a double six, while the problem of points asks how to divide the stakes if a game of dice is incomplete. They solved the problem of points for a two player game, but did not develop powerful enough mathematical methods to solve it for three or more players.

Terminology

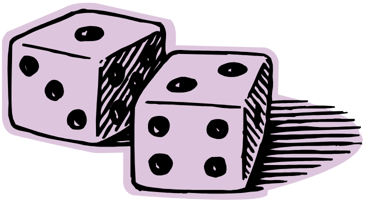
- **Definition:** A **random experiment** is a process or activity which produces a number of possible outcomes. The outcomes cannot be predicted with absolute certainty.
  - **Example 1**: Flip two coins and observe the possible outcomes of heads and tails.

Further Examples of Experiments

- **Example 2:** Select two marbles without replacement from a bag containing 1 white, 1 red and 2 green marbles.



- **Example 3:** Roll two dice and observe the sum of the points on the top faces.



- All of the above are considered **experiments.**

More Terminology

- The **sample space** is a list of all possible outcomes of the experiment. The outcomes must be mutually exclusive and exhaustive. **Mutually exclusive** means they are distinct and non-overlapping. **Exhaustive** means including all possibilities.
- An **event** is a subset of the sample space. An event can be classified as a **simple event** (a subset which contains exactly one element of the sample space) or a **compound event** (a subset of two or more elements).

Example

- The **experiment** is to select a card from an ordinary deck of playing cards (no jokers).
- The **sample space** consists of the 52 cards, 13 of each suit. We have 13 clubs, 13 spades, 13 hearts and 13 diamonds.
- An example of a **simple** event is that the selected card is the two of clubs.
- An example of a **compound event** is that the selected card is red (there are 26 red cards and so there are 26 simple events comprising the compound event).

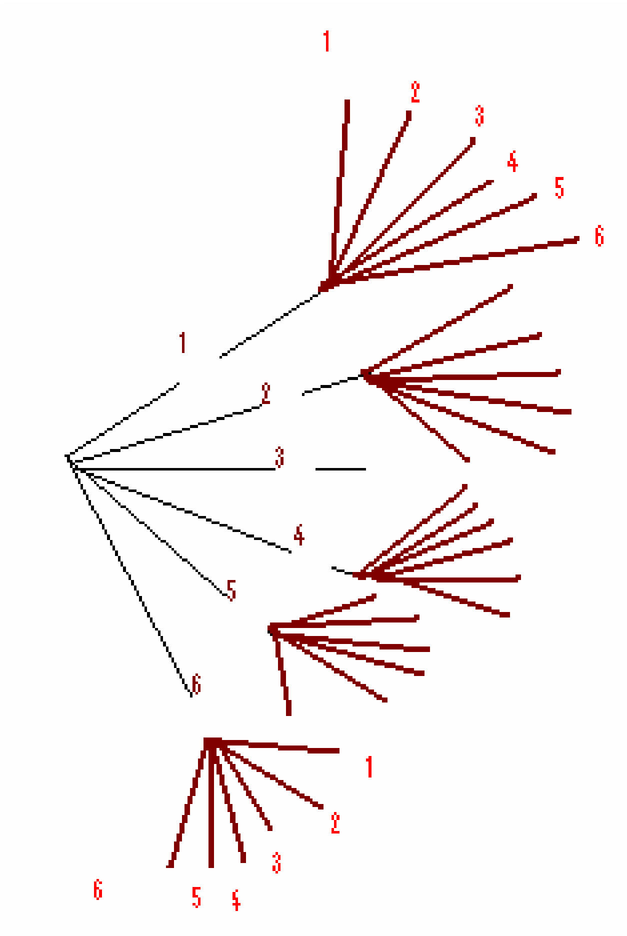
Example 2
Solution

- The experiment is to select a driver randomly from all drivers in the age category of 18-25. Identify the sample space, give an example of a simple event and a compound event.
- **Solution:** The sample space is the set of all drivers age 18-25. A simple event is “Joe Smith”. A compound event is “all drivers age 23”.

Example

- Roll two dice. Describe the sample space of this event.
  The answer to this problem will be given on the next two slides.

Solution



You can use a tree diagram to determine the sample space of this experiment. There are six outcomes on the first die {1,2,3,4,5,6} and those outcomes are represented by six branches of the tree starting from the “tree trunk”. For each of these, there are six outcomes for the second die, represented by the brown branches. Therefore there are 6*6=36 outcomes. They are listed on the next slide.

Solution (continued)

Sample space of all possible outcomes when two dice are tossed:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Quite a tedious project !!

Probability

- There are two basic approaches to assigning probabilities to events:
  - **Theoretical -** This uses basic assumptions and a reasoning process.
  - **Empirical -** This type of probability is determined by experimentation. We assign probabilities to events based on the results of actual experiments.
- We will first consider the theoretical approach.

Probability of an Event

- The **equally likely assumption** means that all simple events have the same probability. If there are n simple events, they each have probability $1/n$. This is a **theoretical probability**.
- The **probability of an event** is the sum of the probabilities of the simple events that constitute the event.
- Under the equally likely assumption, the probability of a compound event is the number of elements in the event, divided by the number of events of the sample space.

$$P(E) = \frac{n(E)}{n(S)}$$

Theoretical Probability Example (continued)

- Find the probability of a sum of 7 when two dice are rolled.
- **Solution:** Look back at the slide that gives the sample space. The number of events of the sample space is 36.
- 6 of these 36 possibilities give a sum of 7:

  {(1,6), (6,1), (2,5), (5,2), (4,3) (3,4)}

  The outcome (1,6) is different from (6,1) in that (1,6) means a one on the first die and a six on the second die, while a (6,1) outcome represents a six on the first die and one on the second die. The answer is

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Empirical Probability

- If we conduct an experiment n times and event E occurs with frequency $f(E)$, then the ratio $f(E)/n$ is called the **relative frequency** or **approximate empirical probability** of the occurrence of event E in n trials.
- Empirical probability relies upon the long run relative frequency of an event. For example, out of the last 1000 statistics students, 150 of the students received an A. Thus, the empirical probability that a student receives an A is 0.15.
- **Example:** The batting average of a major league ball player can be interpreted as the probability that he gets a hit on a given time at bat.

Empirical Probability Example

The manager of a records store has kept track of the number of CDs for a particular title sold per day. On the basis of this information, the manager produced the following list of the number of daily sales:

Number of CDs	Probability
0	0.08
1	0.17
2	0.26
3	0.21
4	0.18
5	0.10

Empirical Probability Example (continued)

1. Define the experiment as the number of CD's sold tomorrow. What is the sample space?
2. What is the probability that the number of CD's sold is greater than 3?
3. What is the probability of selling exactly 5 CD's?
4. What is the probability that the number of CD's sold is between 1 and 5 inclusive?
5. What is the probability of selling 6 CD's?

Empirical Probability Example (continued)

1. Define the experiment as the number of CD's sold tomorrow. What is the sample space? {0,1,2,3,4,5}
2. What is the probability that the number of CD's sold is greater than 3? 0.18 + 0.10 = 0.28
3. What is the probability of selling exactly 5 CD's? 0.10
4. What is the probability that the number of CD's sold is between 1 and 5 inclusive? 0.92
5. What is the probability of selling 6 CD's? 0

Theoretical versus Empirical Probability

What does it mean to say that the probability of getting a sum of 7 upon rolling two dice is 1/6? This is the **long-range probability** or **theoretical probability**, based upon the assumption that all possible rolls are equally likely. It is calculated without doing an experiment.

The theoretical probability of an event should be close to the **experimental probability** if the experiment is repeated a great number of times.

If you rolled two dice a great number of times, in the long run the proportion of times a sum of 7 came up would be approximately 1/6.

Some Properties of Probability

$$0 \leq P(E) \leq 1$$

$$P(E_1) + P(E_2) + P(E_3) + \ldots = 1$$

- The first property states that the probability of any event will always be a number between 0 and 1 (inclusive). If $P(E) = 0$, we say that event E is an **impossible event.** If $P(E) = 1$, we call event E a **certain event**. Some have said that there are two certainties in life: death and taxes.
- The second property states that the sum of the probabilities of all simple events of the sample space must equal 1.

Steps for Finding the Probability of an Event *E*

- **Step 1.** Set up an appropriate sample space *S* for the experiment.
- **Step 2.** Assign acceptable probabilities to the simple events in *S* (theoretically or empirically)
- **Step 3.** To obtain the probability of an arbitrary event *E*, add the probabilities of the simple events in *E*.

Example (continued)

- **Example:** Toss two coins. Find the probability of at least one head appearing.
- **Solution:** "At least one head" is interpreted as "one head or two heads." This is a theoretical probability.
  - **Step 1:** Find the sample space: {HH, HT, TH, TT}. There are four possible outcomes.
  - **Step 2:** How many outcomes of the event "at least one head"? Answer: Three, which are { HH, HT, TH}
  - **Step 3:** Use $P(E) = \frac{n(E)}{n(S)}$ = ¾ = 0.75 = 75%

Simulation and Empirical Probabilities

We can use output from the random number feature of a graphing calculator to simulate 100 rolls of two dice. Determine the empirical probabilities of the following events, and compare them with the theoretical probabilities:

(A) E_1 = a sum of 7 turns up

(B) E_2 = a sum of 11 turns up

Solution to Empirical Problem

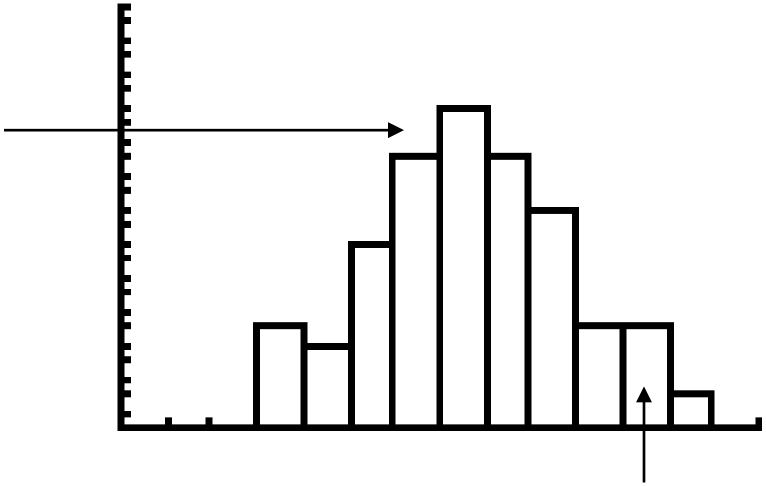
A graphing calculator can be used to select a random integer from 1 to 6. Each of the six integers in the given range is equally likely to be selected. Therefore, by selecting a random integer from 1 to 6 and adding it to a second random integer from 1 to 6, we simulate rolling two dice and recording the sum by using the first command below.

```
randInt(1,6)+ran
dInt(1,6)
               6
randInt(1,6,100)
+randInt(1,6,100
)→L1
{6 6 3 12 9 6 6…
```

The second command in this figure simulates 100 rolls of two dice; the sums are stored in list L_1.

Solution to Empirical Problem (continued)

An outcome of 7 is the highest bar; it occurs 20 times out of 100. Therefore the empirical probability of E_1 is 20/100 = 0.20; the theoretical probability is 6/36 = 0.167



Note: If you simulate this experiment on your graphing calculator, you should not expect to get the same empirical probabilities.

The empirical probability of E_2 is 6/100 = 0.06; the theoretical probability of E_2 is 2/36 = 0.056