8.2. Union, Intersection, and Complement of Events

Goals:

1. Determine the union and intersection of events
2. The Addition Rule
3. Determine the complement of an event
4. Determine the odds in favor and odds against an event
5. Solve applications.

E.g. Toss 2 fair coins

Sample Space $S = \{HH, HT, TT, TH\}$

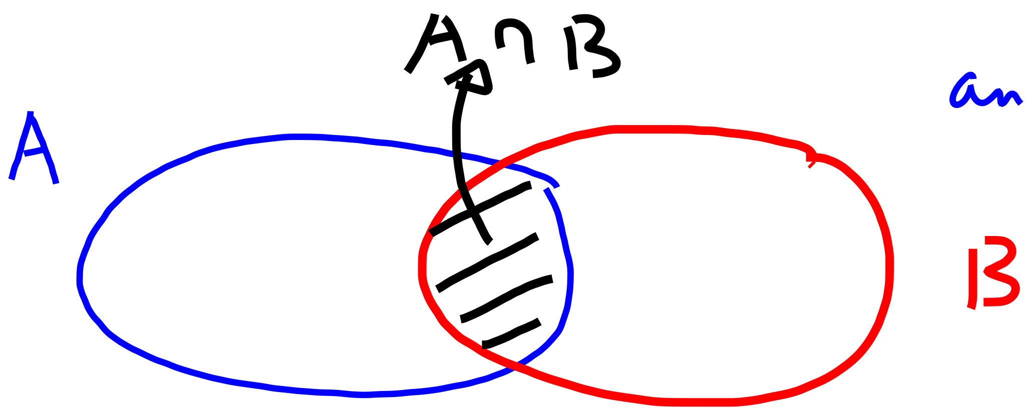
Event A = gets at least 1 head = $\{HH, HT, TH\}$

Event B = gets exactly 1 tail = $\{HT, TH\}$

Intersection of A and B = $\{HT, TH\}$

$$A \cap B = \{HT, TH\}$$

Describe in words = gets exactly 1 T and gets at least 1 H

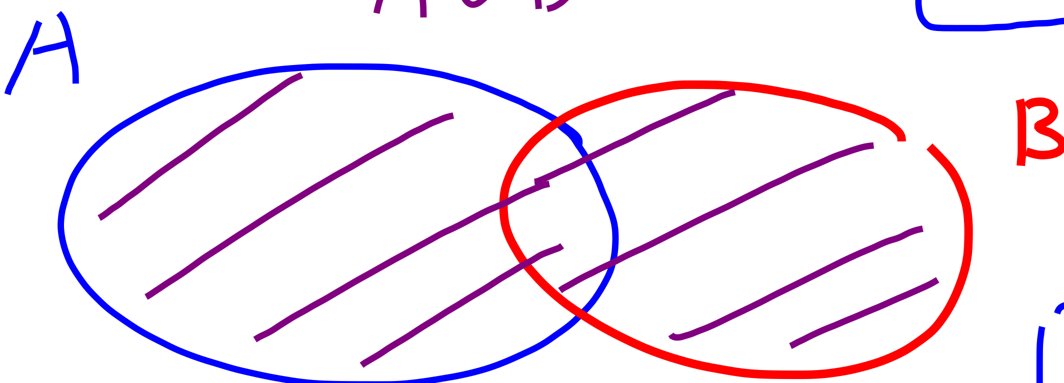


Union of A and B = {HT, TH, HH}

$A \cup B = \{HT, TH, HH\}$

Describe $A \cup B$ in words = gets at least 1 H $\boxed{\text{or}}$ gets exactly 1 T.

$A \cup B$



$n(A \cup B) =$

$$\boxed{n(A) + n(B) - n(A \cap B)}$$

The addition rule (how to find the probability of the union of 2 events }

$A \cup B = \{HT, TH, HH\}$.

$\boxed{P(A \cup B)} = \dfrac{n(A \cup B)}{n(S)} = \dfrac{3}{4}$

Note: $P(A) = \dfrac{3}{4}$; $P(B) = \dfrac{2}{4}$; $P(A \cap B) = \dfrac{2}{4}$

$\boxed{P(A) + P(B) - P(A \cap B)} = \dfrac{3}{4} + \dfrac{2}{4} - \dfrac{2}{4}$

$= \dfrac{3}{4}$.

Addition Rule

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

E.g. Experiment: Draw a card from a 52 card deck

Q: Probability that the card is a jack or a club.

A: gets a jack. $P(A) = \frac{4}{52}$

B: gets club. $P(B) = \frac{13}{52}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cap B$ = get a jack of clubs. $P(A \cap B) = \frac{1}{52}$

$P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

E.g.: Experiment: Toss 2 dice.

S: has 36 outcomes.

Q: Probability that you get a sum greater than 8 **or** doubles.

A: gets a sum > 8.

B = gets doubles

$A = \{(3,6), (4,5), (5,5), (6,3), (5,4), (6,6), (4,6), (6,4), (6,5), (5,6)\}$.

$\boxed{P(B) = \frac{6}{36}}$ $\boxed{P(A) = \frac{10}{36}}$

$\boxed{P(A \cap B) = \frac{2}{36}}$

$P(A \cup B) =$

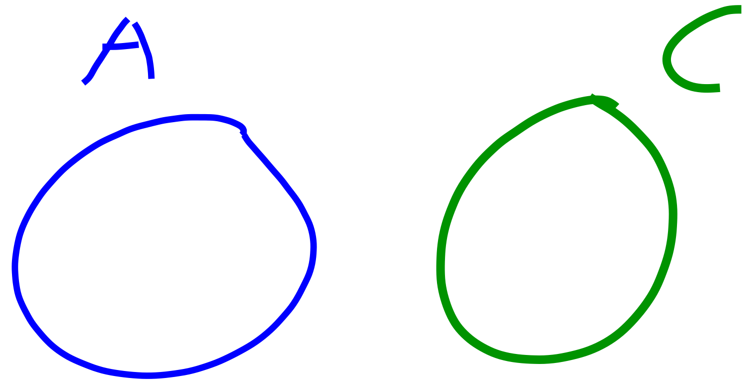
$\frac{6}{36} + \frac{10}{36} - \frac{2}{36} = \frac{14}{36} = \frac{7}{18}$.

Mutually Exclusive Events

A = gets at least 1 head = $\{HT, TH, HH\}$

C = get exactly 2 tails = $\{TT\}$

$$A \cap C = \emptyset$$



A and C are called mutually exclusive events.

(their intersection is the empty set)

$$P(A \cup C) = P(A) + P(C) - \cancelto{0}{P(A \cap C)}$$

$$P(A \cup C) = P(A) + P(C)$$

Complement of an event.

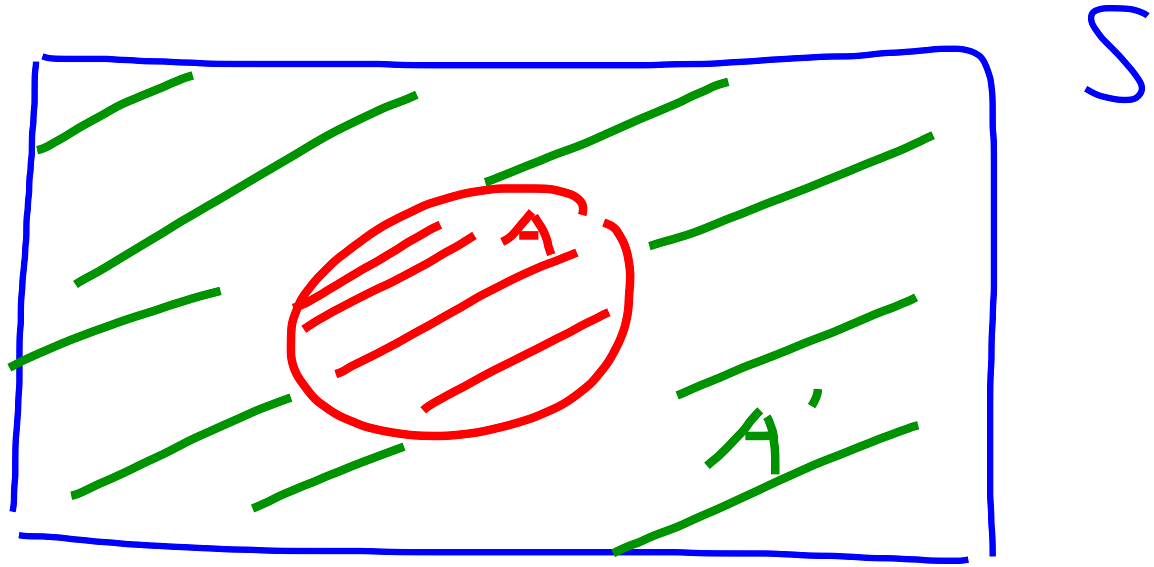
A = get at least 1 head = $\{HT, TH, HH\}$

A': the complement of A in S: elements in S

but not in A

$A' = \{TT\}$. Describe A' in words: get no

heads



$$P(A') = \frac{1}{4}$$

$$= 1 - \frac{3}{4} = 1 - P(A)$$

$$\boxed{P(A') = 1 - P(A)}$$

E.g. Toss 2 dice.

Probability that the # of points on each dice will not be the same?

A = # of points are not the same

A' = # of points are the same. $P(A') = \frac{6}{36}$.

$$P(A) = 1 - P(A') = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

Odds in favor of an event and odds against an event.

$A = \{HT, TH, HH\}$.

odds in favor of A is 3 to 1 $= 3 = \dfrac{P(A)}{P(A')} = \dfrac{\frac{3}{4}}{\frac{1}{4}}$

odds against A is 1 to 3 $= \dfrac{P(A')}{P(A)}$.

In general, to find the odds in favor of any event E; the find $\dfrac{P(E)}{P(E')}$. odds against: $\dfrac{P(E')}{P(E)}$.

Eg. Find the odds in favor of getting a sum of 7 when 2 dice are tossed.

Find the odds against this.

A = get a sum of 7. $\quad P(A) = \frac{6}{36}$

A' = get a sum $\neq 7$. $\quad P(A') = \frac{30}{36}$.

odds in favor of A = $\dfrac{\frac{6}{36}}{\frac{30}{36}} = \frac{6}{30} = \frac{1}{5}$.

odds against A : $\frac{5}{1}$.