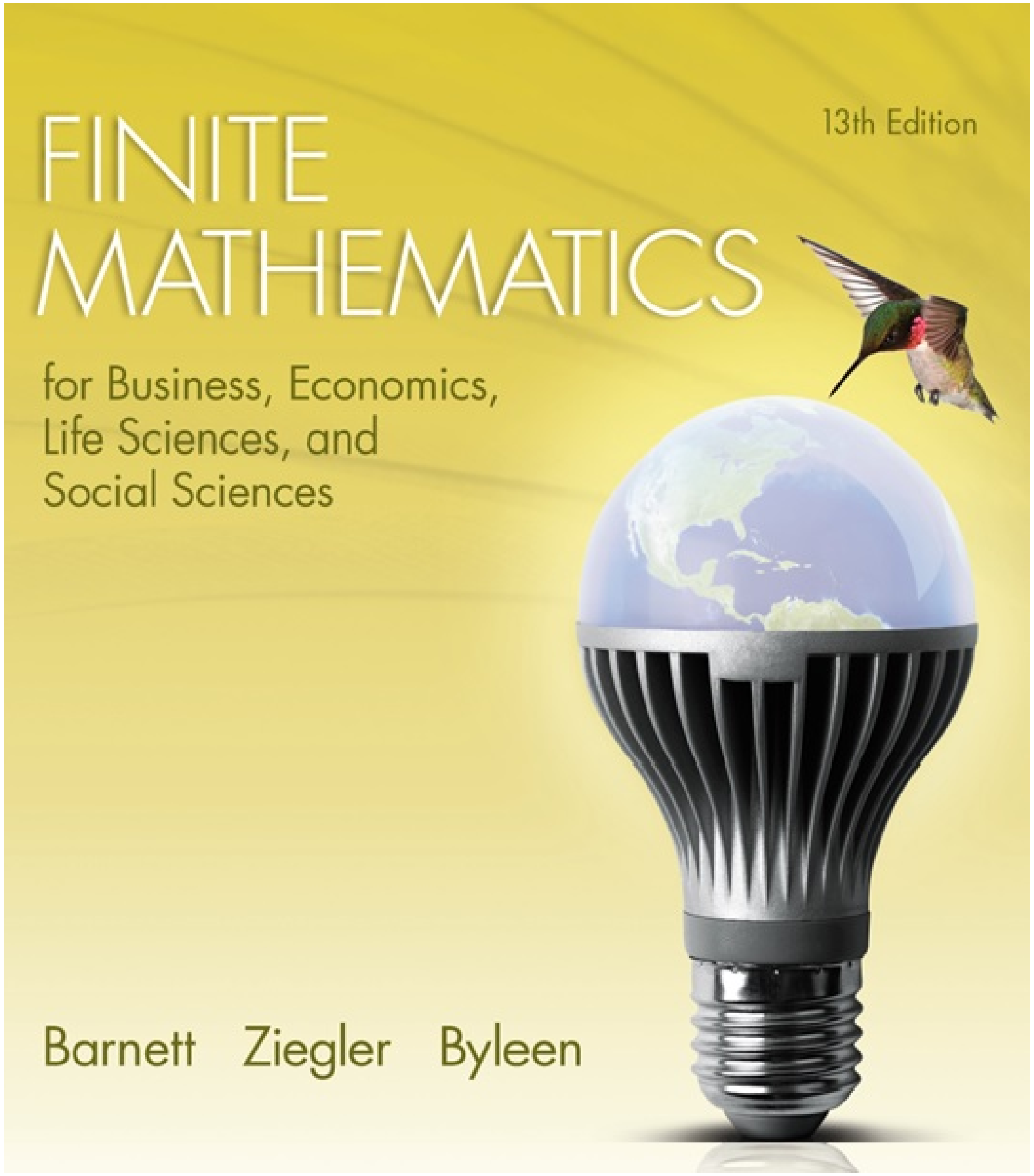


Chapter 8

Probability

Section 2

Union, Intersection, and Complement of Events, Odds

Objectives for Section 8.2

Union, Intersection, Complement of Events; Odds

- The student will be able to determine the union and intersection of events.
- The student will be able to determine the complement of an event.
- The student will be able to determine the odds of an event.
- The student will be able to solve applications of empirical probability.

Union, Intersection, Complement of Events; Odds

- In this section, we will develop the rules of probability for **compound events (**more than one simple event) and will discuss probabilities involving the **union** of events as well as **intersection** of two events.

Union and Intersection of Events

If A and B are two events in a sample space S, then the **union** of A and B, denoted by $A \cup B$, and the **intersection** of A and B, denoted by $A \cap B$, are defined as follows:

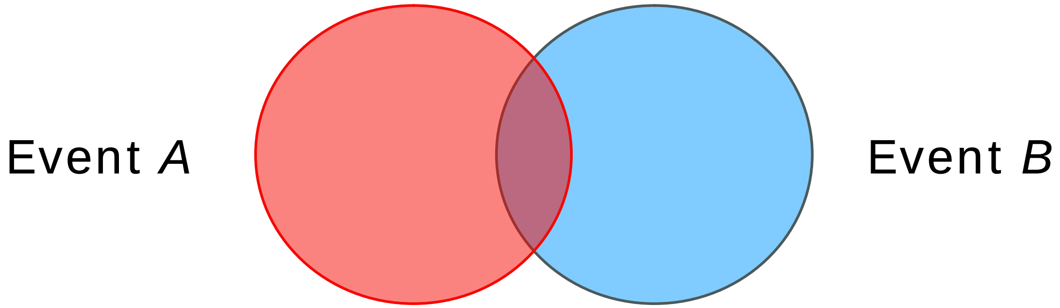
$$A \cup B = \{e \in S \mid e \in A \text{ or } e \in B\}$$

$$A \cap B = \{e \in S \mid e \in A \text{ and } e \in B\}$$

Number of Events in the Union

Sample space S

The number of events in the union of A and B is equal to the number in A plus the number in B minus the number of events that are in both A and B.



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Addition Rule

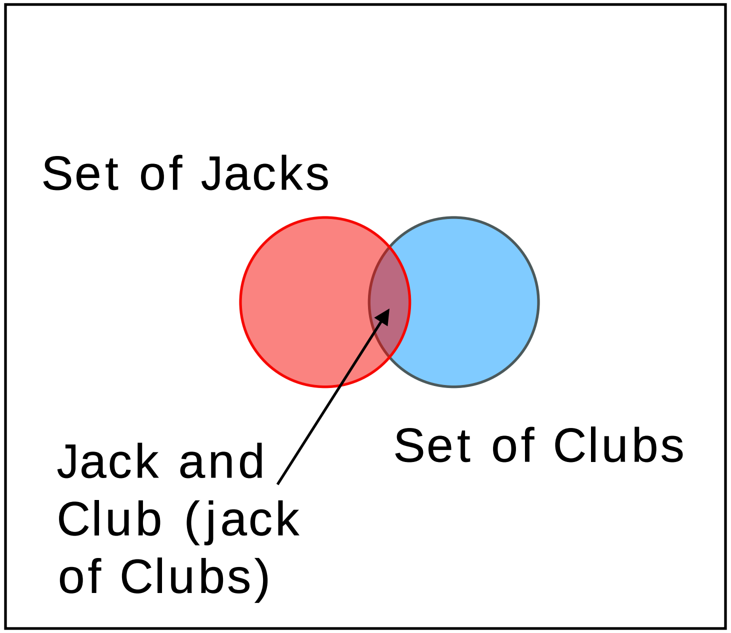
- If you divide both sides of the equation by $n(S)$, the number of simple events in the sample space, we can convert the equation to an equation of probabilities:

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \rightarrow$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example of Addition Rule

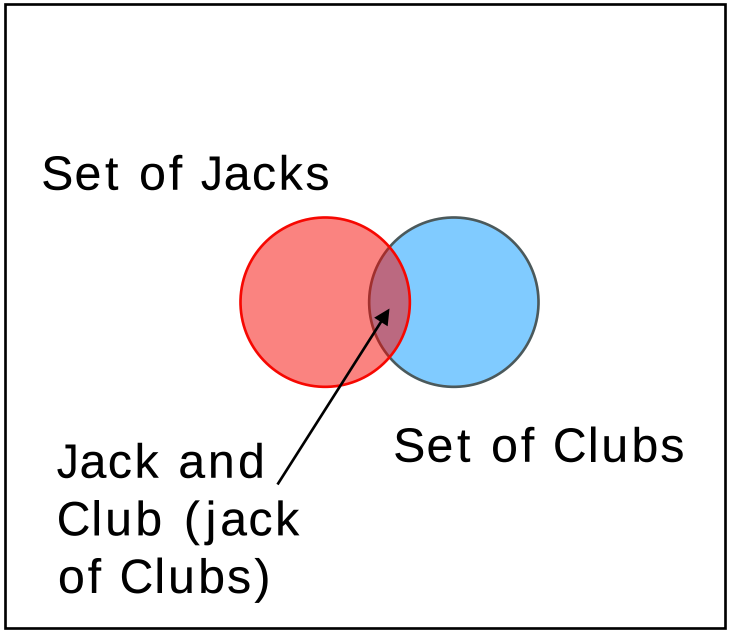
- A single card is drawn from a deck of cards. Find the probability that the card is a jack or club.



Example of Addition Rule

- A single card is drawn from a deck of cards. Find the probability that the card is a jack or club.

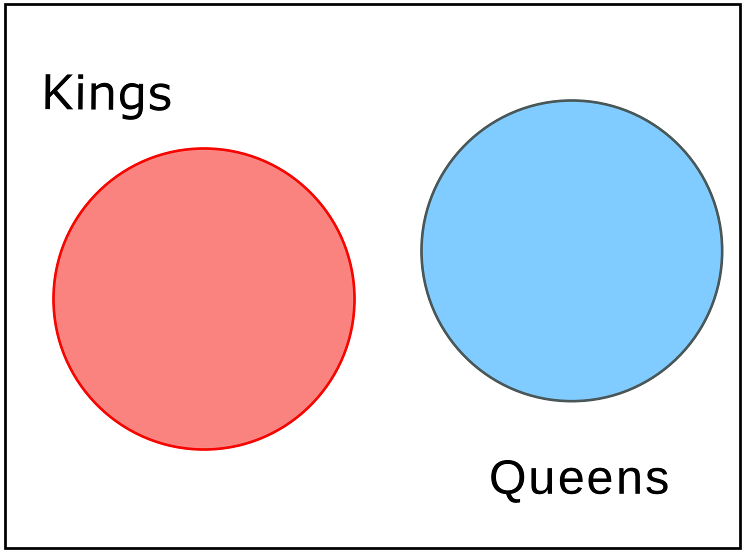
$$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$



$$P(J \text{ or } C) = P(J) + P(C) - P(J \text{ and } C)$$

Union of Mutually Exclusive Events

- A single card is drawn from a deck of cards. Find the probability that the card is a king or a queen.
- The events *King* and *Queen* are **mutually exclusive**. They cannot occur at the same time. So the probability of *King and Queen* is zero.



$$4/52 + 4/52 – 0 = 8/52 = 2/13$$

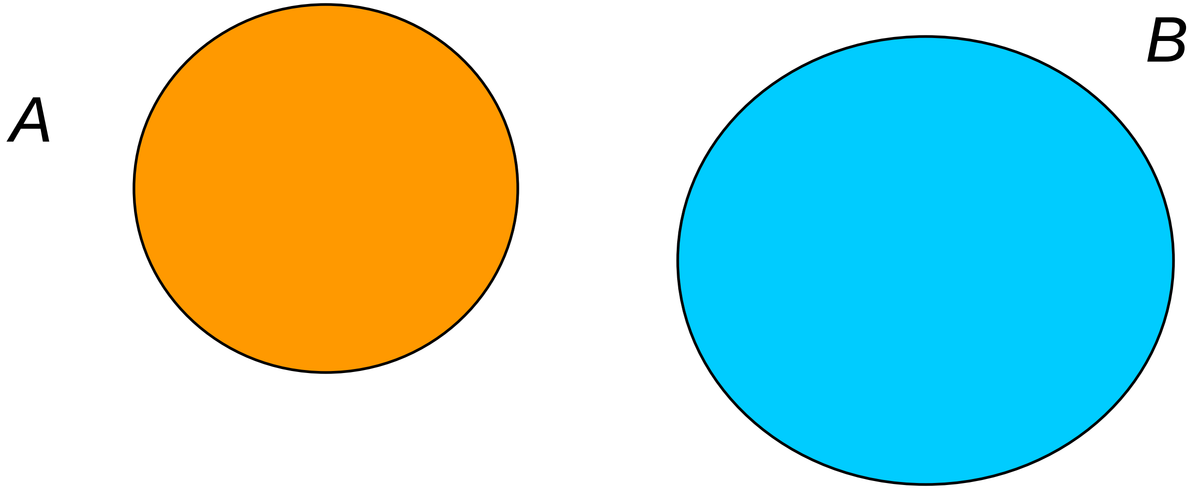
$$P(K \cup Q) = p(K) + P(Q) - p(K \cap Q)$$

Mutually Exclusive Events

If *A* and *B* are mutually exclusive then

$$P(A \cup B) = P(A) + P(B)$$

The intersection of *A* and *B* is the empty set.



Using a Table to List Outcomes of an Experiment

- Three coins are tossed. Assume they are fair coins. Tossing three coins is the same experiment as tossing one coin three times. There are two outcomes on the first toss, two outcomes on the second toss and two outcomes on toss three. Use the multiplication principle to calculate the total number of outcomes: (2)(2)(2) = 8
- Each row of the table consists of a simple event of the sample space. The indicated row, for instance, illustrates the outcome ({heads, heads, tails}, in that order.)

h	h	h
h	h	t
h	t	h
h	t	t
t	h	h
t	h	t
t	t	h
t	t	t

Using a Table (continued)

h	h	h
h	h	t
h	t	h
h	t	t
t	h	h
t	h	t
t	t	h
t	t	t

To find the probability of at least two tails, we mark each row (outcome) that contains two tails or three tails and divide the number of marked rows by 8 (number in the sample space) Since there are four outcomes that have at least two tails, the (theoretical) probability is 4/8 or ½ .

Example Solution

Two dice are tossed. What is the probability of a sum greater than 8, or doubles?

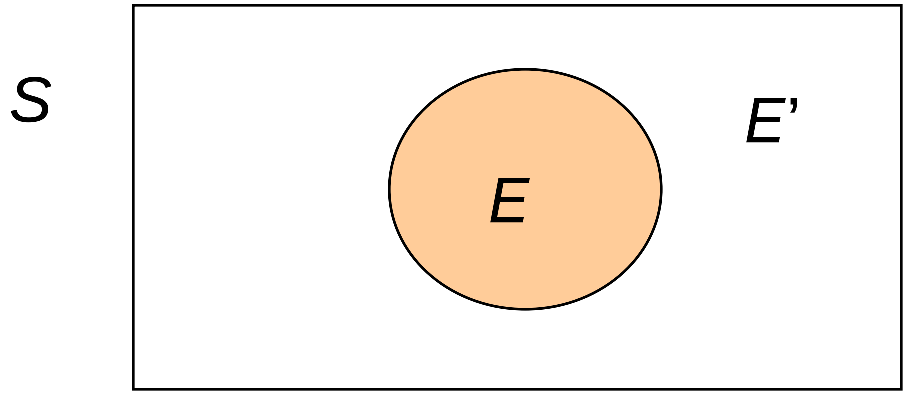
$P(S > 8 \text{ or doubles}) = P(S > 8) + P(\text{doubles}) – P(S > 8 \text{ and doubles}) = 10/36 + 6/36 – 2/36 = 14/36 = 7/18.$

(1,1), (1,2), (1,3), (1,4), (1,5) (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Complement of an Event

Suppose that we divide a finite sample space $S = \{e_1, \ldots, e_n\}$ into two subsets E and E' such that $E \cap E' = \varnothing$. That is, E and E' are mutually exclusive, and $E \cup E' = S$.

Then E' is called the **complement of *E*** relative to S. Thus, E' contains all the elements of S that are not in E, as below.



Complement Rule

Many times it is easier to compute the probability that *A* won't occur, then the probability of event *A*:

$$p(A) + p(A') = 1 \rightarrow p(A') = 1 - p(A)$$

- **Example:** What is the probability that when two dice are tossed, the number of points on each die will **not** be the same?

Complement Rule

Many times it is easier to compute the probability that *A* won't occur, then the probability of event *A*:

$$p(A) + p(A') = 1 \rightarrow p(A') = 1 - p(A)$$

- **Example:** What is the probability that when two dice are tossed, the number of points on each die will **not** be the same?
- This is the same as saying that doubles will not occur. Since the probability of doubles is 6/36 = 1/6, then the probability that doubles will not occur is 1 – 6/36 = 30/36 = 5/6.

Odds

- In certain situations, such as the gaming industry, it is customary to speak of the **odds in favor** of an event *E* and the **odds against** *E*. The definitions are

- **Odds in favor of event *E* =** $\frac{p(E)}{p(E')}$

- **Odds against *E* =** $\frac{p(E')}{p(E)}$

Odds (continued)

- **Example:** Find the odds in favor of rolling a seven when two dice are tossed.
- **Solution**: The probability of a sum of seven is 6/36. The probability of the complement is 30/36. So,

$$\frac{p(E)}{p(E')} = \frac{\frac{6}{36}}{\frac{30}{36}} = \frac{6}{30} = \frac{1}{5}$$

Empirical Probability Example

The data on the next slide was obtained from a random survey of 1,000 residents of a state. The participants were asked their political affiliations and their preferences in an upcoming gubernatorial election

(D = Democrat, R = Republican, U = Unaffiliated.)

If a resident of the state is selected at random, what is the empirical probability that the resident is not affiliated with a political party or has no preference?

Empirical Probability Example (continued)

	D	R	U	Totals
Candidate A	200	100	85	385
Candidate B	250	230	50	530
No Preference	50	20	15	85
Totals	500	350	150	1,000

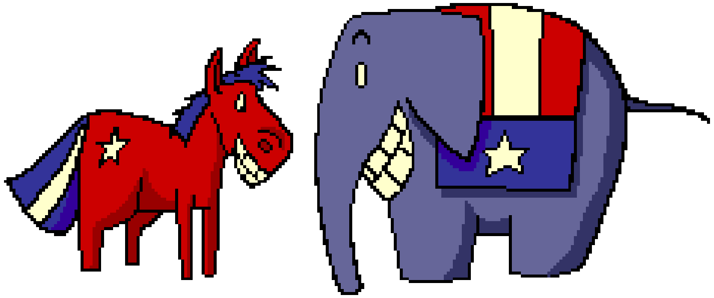
Empirical Probability Example Solution

P(unaffiliated or has no preference) =

P(unaffiliated) + P(has no preference)

– P(unaffiliated and has no preference) =

$$\frac{150}{1000} + \frac{85}{1000} - \frac{15}{1000} = \frac{220}{1000} = 0.22$$



Law of Large Numbers

In mathematical statistics, an important theorem called the **law of large numbers** is proved. It states that the approximate empirical probability can be made as close to the actual probability as we please by making the sample sufficiently large.