8.3. Conditional Probability

Goals: 1. Calculate Conditional Probability.

2. Use conditional probability to calculate the probability of the intersection of 2 events.

3. Solve applications by constructing probability trees.

4. Understand independent events.

Conditional probability.

Problem: find the probability that a randomly selected person in the U.S. has lung cancer

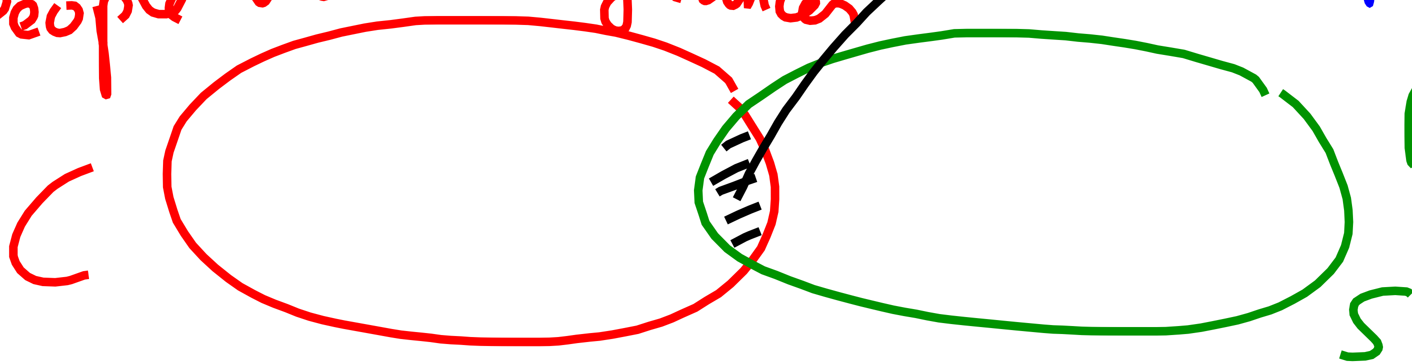
$$P(C) = \frac{n(C)}{n(T)}$$

Change problem:

find probability that a person has lung cancer given that the person smokes.

$$P(C|S) \longrightarrow \text{conditional probability}$$

$C \cap S$ probability



$$P(C \mid S) = \frac{n(C \cap S)}{n(S)}$$

In general, the probability of event A given that event B already happened is :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

E.g

	Undergrads	Grads	Total
Nursing	53	47	100
Engineering	37	13	50
Total	90	60	150

① $P(N) = \dfrac{100}{150}$

② $P(G) = \dfrac{60}{150}$

③ $P(N \mid U) = \dfrac{53}{90}$

$P(N \cap U) = \dfrac{53}{150}$

$P(U) = 90/150$

Note. $P(N \mid U) = \dfrac{P(N \cap U)}{P(U)}$

Using Conditional Probability to calculate $P(A \cap B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \longrightarrow P(A \cap B) = P(B)\, P(A|B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \longrightarrow P(A \cap B) = P(A) \cdot P(B|A)$$

$$\longrightarrow P(A \cap B) = P(B) \cdot P(A|B)$$

$$= P(A) \cdot P(B|A)$$

E.g. Draw 2 cards from a 52 card deck without replacement.

Q. Probability that you draw 2 clubs in succession.

C_1 = get a club in 1^{st} draw

C_2 = get a club in 2^{nd} draw

$$P(C_1 \cap C_2) = \underbrace{P(C_1)}_{\frac{13}{52}} \underbrace{P(C_2 \mid C_1)}_{\frac{12}{51}}$$

$$\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{4} \cdot \frac{4}{17} = \frac{1}{17}$$

Solve problems using conditional probability and probability trees.

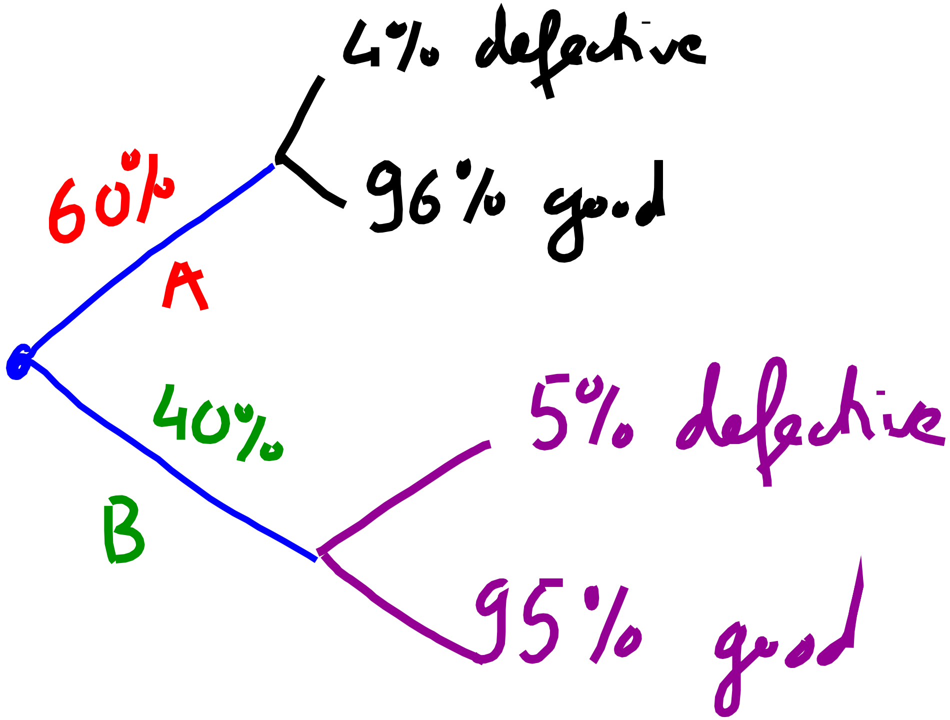
---

2 machines
- A : produces 60% of items
- B : 40% of items

A : 4% defective items.

B : 5% defective items.

Choose an item randomly. What is the probability that it is defective?



$$P(D) = ?$$

$$P(D) = P(A \cap D) + P(B \cap D)$$

$$= P(A) \cdot P(D|A) + P(B) \cdot P(D|B)$$

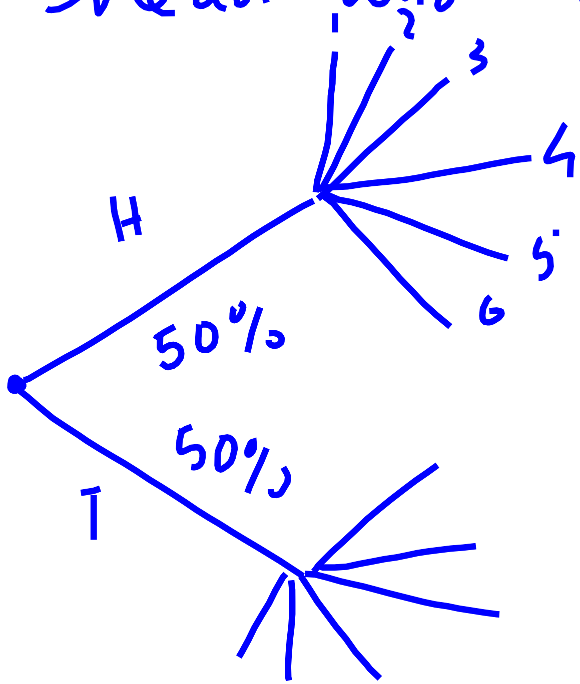
$$= (0.6) \cdot (0.04) + (0.4)(0.05)$$

$$= 0.044$$

Ex.

Toss a coin and roll a dice.

Find probability that the coin comes up head and the dice comes up 3.



$$P(H \cap 3)$$

$$= P(H) \cdot P(3 \mid H)$$

$$= \left(\frac{1}{2}\right) \cdot \left(\frac{1}{6}\right) = \frac{1}{12}$$

$$= 0.083.$$

Independent events.

A and B are independent if:

$$P(B \mid A) = P(B)$$

$$P(A \mid B) = P(A)$$

$$P(A \cap B) = P(B) \cdot \underbrace{P(A \mid B)}_{P(A)}$$

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$