

8.4 Bayes' Formula

Goals: Solve problems using probability trees and Bayes' formula.

Recall:
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\begin{aligned} P(A \cap B) &= P(B) P(A|B) \\ &= P(A) P(B|A) \end{aligned}$$

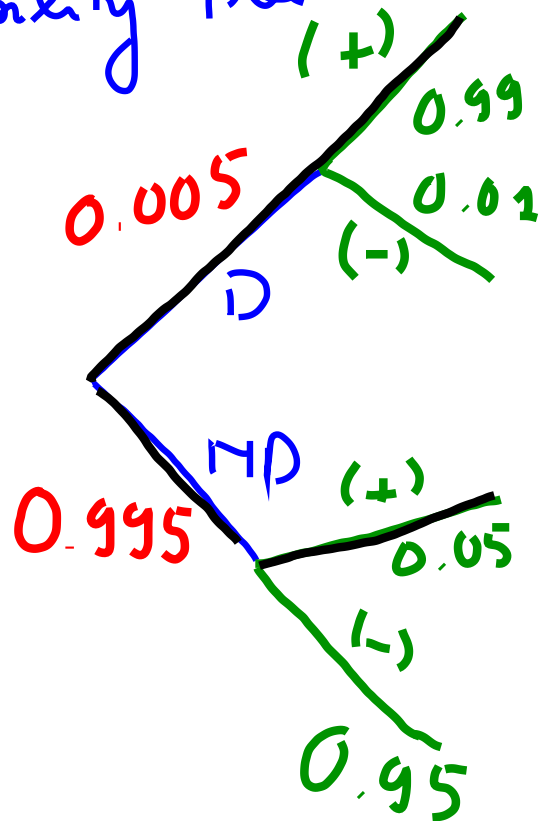
E.g. Rare disease: Found in 0.5% of population.

Blood test: 99% accurate if the disease is present
5% (+) result if the disease is not present

Q: If one gets a (+) result, what is the probability that the person has the disease.

$$(0.005) \cdot (0.99)$$

Probability tree



Question:

$$P(D|+) = ?$$

Known:

$$P(D) = 0.005$$

$$P(ND) = 0.995$$

$$P(+|D) = 0.99$$

$$P(-|D) = 0.01$$

$$P(+|ND) = 0.05 ; P(-|ND) = 0.95$$

$$P(D|+) = \frac{P(D \cap +)}{P(+)}$$

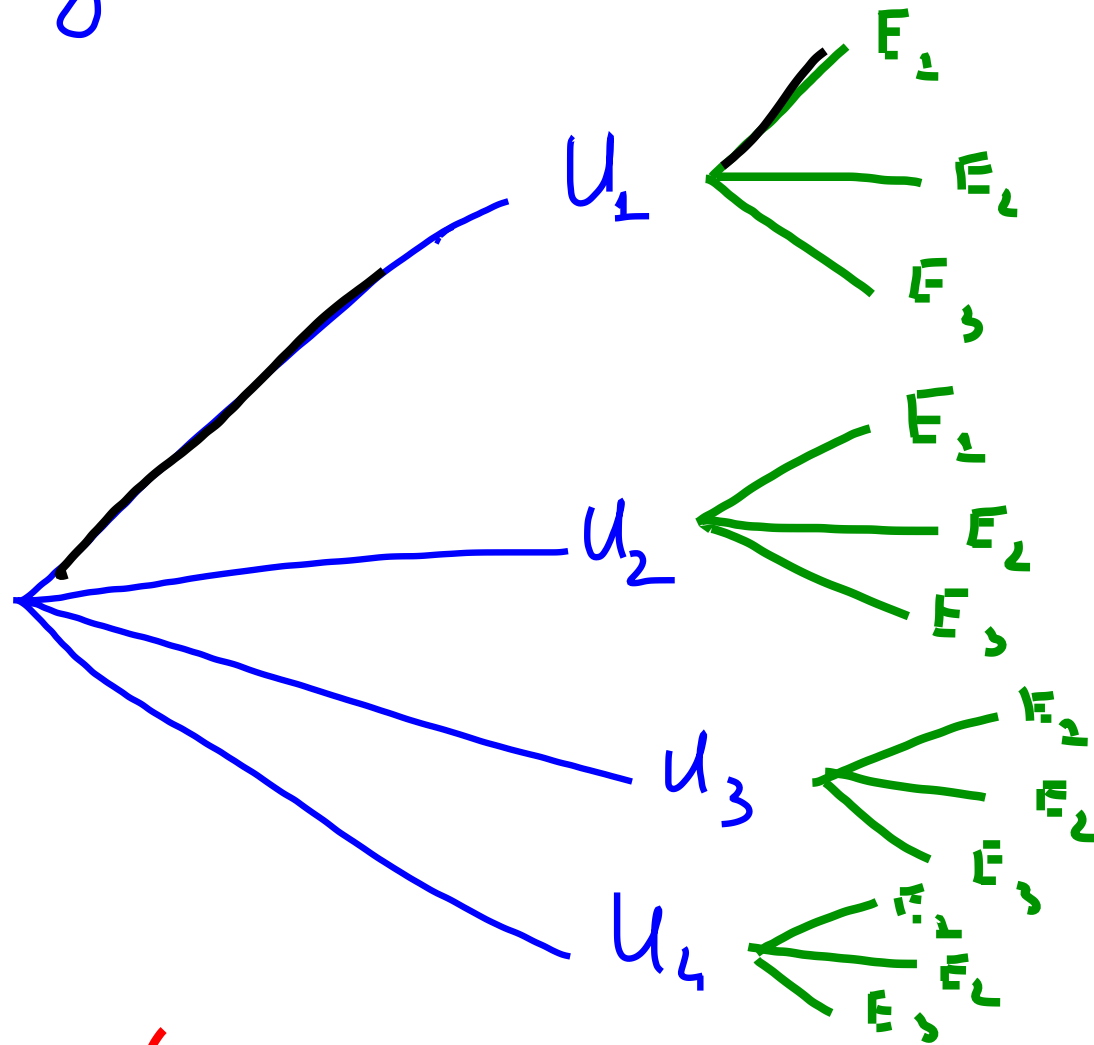
$$\begin{aligned} P(D \cap +) &= P(D) \cdot P(+|D) \\ &= (0.005) \cdot (0.99) = \boxed{0.00495} \end{aligned}$$

$$\begin{aligned} P(+) &= P(D \cap +) + P(\neg D \cap +) \\ &= P(D) \cdot P(+|D) + P(\neg D) \cdot P(+|\neg D) \\ &= (0.005)(0.99) + (0.995)(0.05) \\ &= \boxed{0.0547} \end{aligned}$$

$$P(D|+) = 0.00495 / 0.0547 = 0.0905$$

$$P(\text{NOI-}) = 0.99994 \rightarrow 99.994\%$$

In general, use of probability tree



$$P(U_1 | E_1)$$

=

(Mutually exclusive)

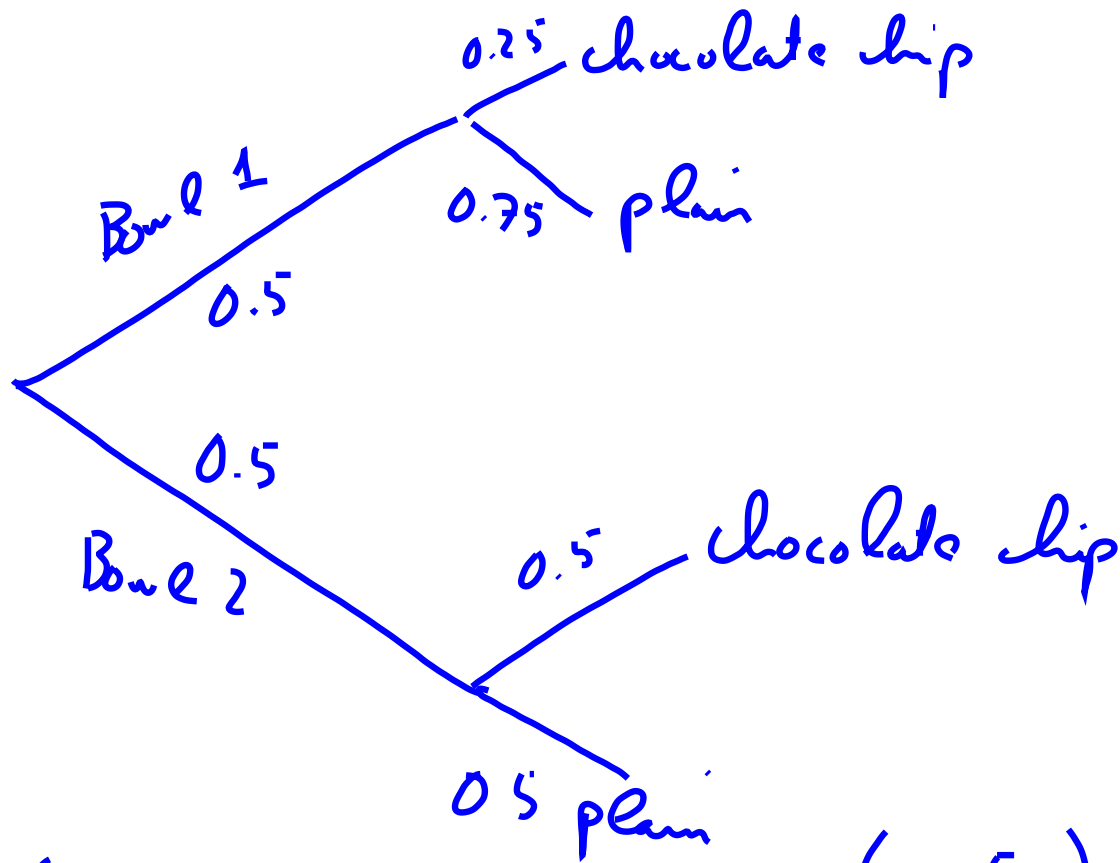
Ex. 2 bowls of cookies

Bowl 1 $\begin{cases} 10 \text{ chocolate chip} \\ 30 \text{ plain} \end{cases}$

Bowl 2 $\begin{cases} 20 \text{ chocolate chip} \\ 20 \text{ plain} \end{cases}$

Billy pick a bowl at random and randomly pick a cookie from that bowl.

If he gets a plain cookie, what is the chance that it comes from bowl 1?



$$P(\text{Bowl 1} | \text{plain}) = \frac{(0.5) \cdot (0.75)}{(0.5)(0.75) + (0.5)(0.5)}$$
$$= 0.6$$

Ex 1. Doctor is called to see a child.

In the neighborhood

90% of such children have the flu

10% _____ have measles

95% of children who have measles have
a rash

5% _____ flu have a rash

Q: If the doctor find a rash, what is the
chance that the child has measles?

$$0.678 = 67.8\%$$

Ex. 2. Companies A, B, C make aircraft emergency locator transmitter (ELT)

A makes 80% of ELT

B — 15% —

C — 5% —

A has 4% rate of defect

B — 6% —

C — 9% —

Q. If a randomly selected ELT is defective, what is the chance it is made by A?

$$P(A | D) = \frac{(0.8) \cdot (0.04)}{(0.8)(0.04) + (0.15) \cdot (0.06) + (0.05) \cdot (0.09)}$$

Bayes' Formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) P(B|A)}{P(B \cap A) + P(B \cap A')}$$

$$= \frac{P(A) \cdot P(B|A)}{P(A) P(B|A) + P(A') \cdot P(B|A')}$$