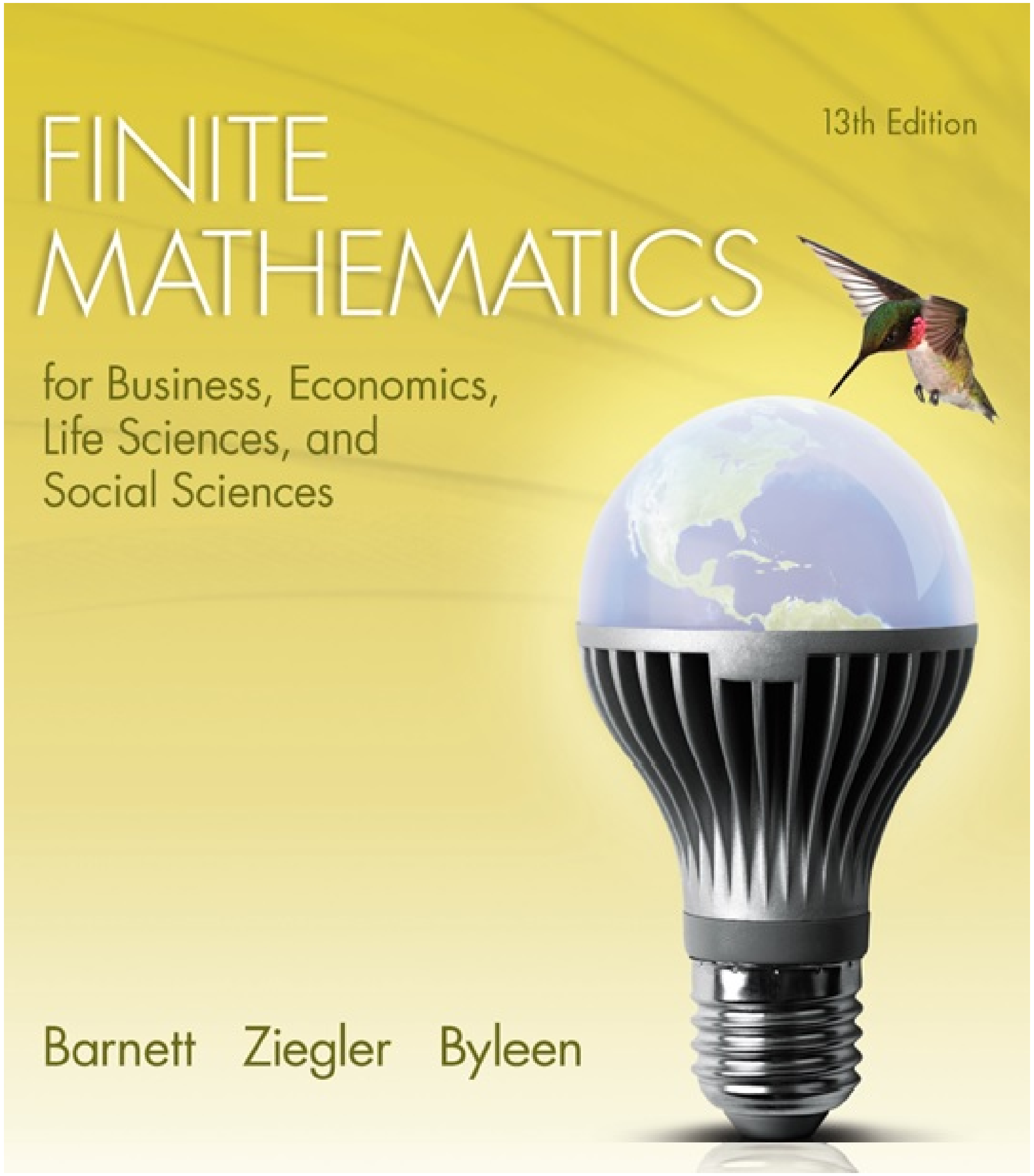


Chapter 8

Probability

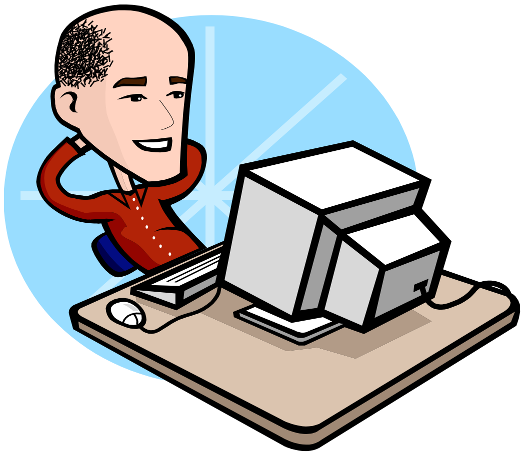
Section 4
Bayes' Formula

Learning Objectives for Section 8.4 Bayes' Formula

- The student will be able to solve problems involving finding the probability of an earlier event conditioned on the occurrence of a later event using Bayes' Formula.
- The student will be able to solve problems of the above type using a probability tree.

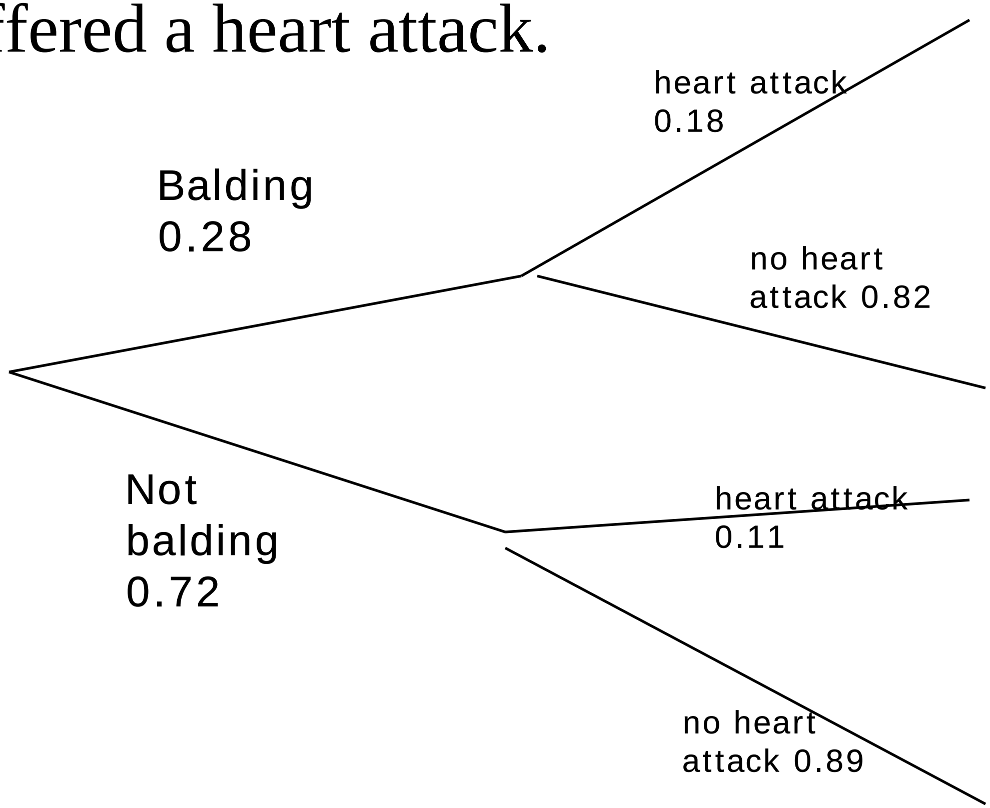
Probability of an Earlier Event Given a Later Event

A survey of middle-aged men reveals that 28% of them are balding at the crown of their head. Moreover, it is known that such men have an 18% probability of suffering a heart attack in the next 10 years. Men who are not balding in this way have an 11% probability of a heart attack. If a middle-aged man is randomly chosen, what is the probability that he is balding, given that he suffered a heart attack? See tree diagram on next slide.



Tree Diagram

- We want $P(B|H)$ = probability that he is balding, given that he suffered a heart attack.



Derivation of Bayes' Formula

$$P(B|H) = \frac{p(B \cap H)}{p(H)}$$

$$p(H) = p(B \cap H) + p(NB \cap H)$$

$$p(H) = p(B)\,p(H|B) + p(NB) \cdot p(H|NB)$$

$$P(B|H) = \frac{p(B \cap H)}{p(B)\,p(H|B) + p(NB) \cdot p(H|NB)}$$

Solution of Problem

$$P(B \mid H) = \frac{p(B \cap H)}{p(B)\,p(H \mid B) + p(NB) \cdot p(H \mid NB)}$$

$$p(B \mid H) = \frac{p(B) \cdot p(H \mid B)}{p(B)\,p(H \mid B) + p(NB) \cdot p(H \mid NB)}$$

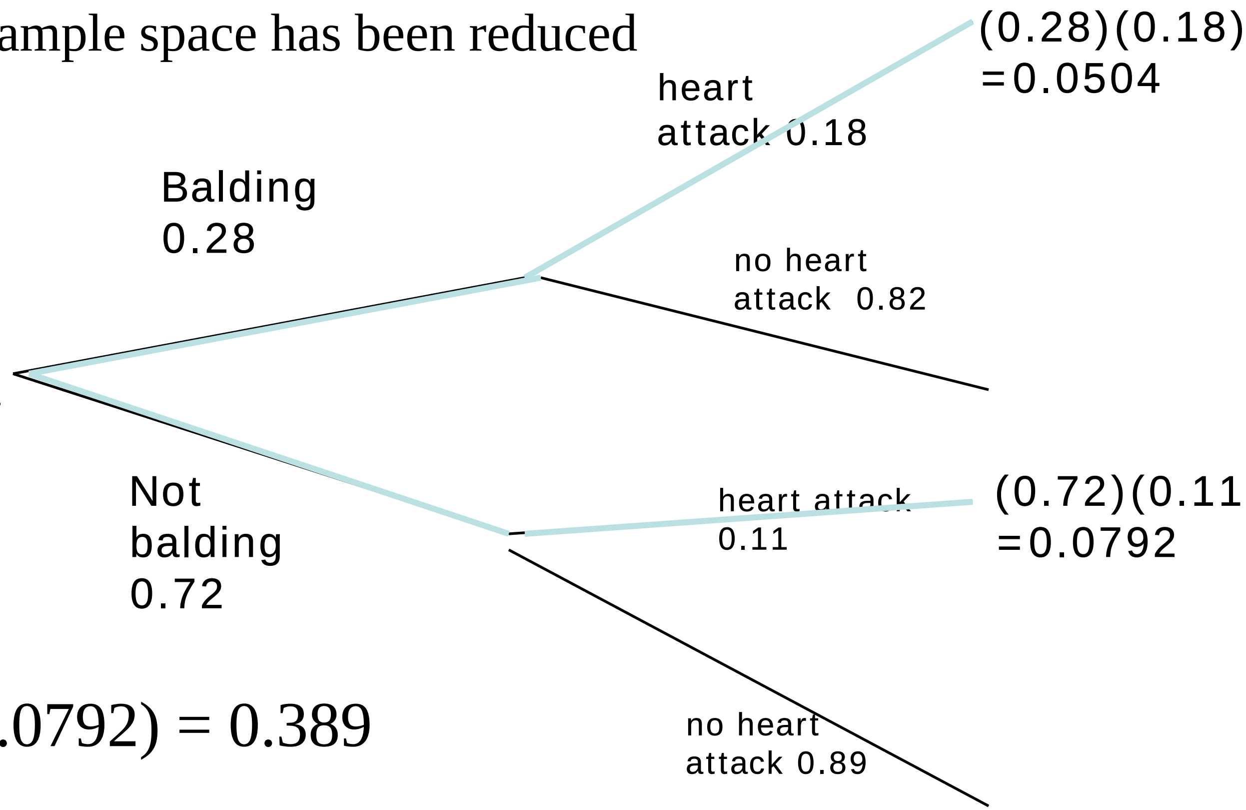
$$p(B \mid H) = \frac{0.28 \cdot (0.18)}{0.28(0.18) + 0.72 \cdot (0.11)} = 0.389$$

Another Method of Solution

Another way to look at this problem is that we know the person had a heart attack, so he has followed one of the two red paths. The sample space has been reduced to these paths. So the

probability that he is balding is

$$\frac{P(B\cap H)}{P(B\cap H)+P(NB\cap H)}$$



which is

0.0504/(0.0504 + 0.0792) = 0.389

Summary of Tree Method of Solution

You do not need to memorize Bayes' formula. In practice, it is usually easier to draw a probability tree and use the following:

Let $U_1, U_2, \ldots U_n$ be n mutually exclusive events whose union is the sample space S. Let E be an arbitrary event in S such that $P(E) \neq 0$. Then

$$P(U_1|E) = \frac{\text{product of branch probabilities leading to } E \text{ through } U_1}{\text{sum of all branch products leading to } E}$$

Similar results hold for $U_2, U_3, \ldots U_n$