8.5 Random Variable, Probability Distribution, and Expected Value.

Goals: (1) Understand the concept of a random variable

(2) Find the probability distribution of a random variable

(3) Find the expected value of a random variable

(4) Solve some applications.

S : sample space.

A random variable is a function or a rule that assigns a numerical value to each outcome in the sample space S.

Eg. Toss coin twice

$$S = \{HH, HT, TH, TT\}$$

X : random variable

$$X(HH) = \$4 \qquad X(TH) = \$1$$

$$X(HT) = \$1 \qquad X(TT) = -\$2$$

Probability distribution.

A probability distribution is a function corresponding to a random variable.

It is defined as:

$$p(x) = P(X = x)$$

Eg. $S = \{HH, HT, TH, TT\}$

$X(HH) = 4$

$X(HT) = X(TH) = 1$

$X(TT) = -2$

x	$p(x)$
-2	$\frac{1}{4}$
1	$\frac{1}{2}$
4	$\frac{1}{4}$

$p(-2) = P(X = -2)$

$p(1) = P(X = 1)$

$p(4) = \frac{1}{2}$

Expected Value of a random variable.

Eg. $-2 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4}$

$$E(x) = \sum x \cdot p(x)$$