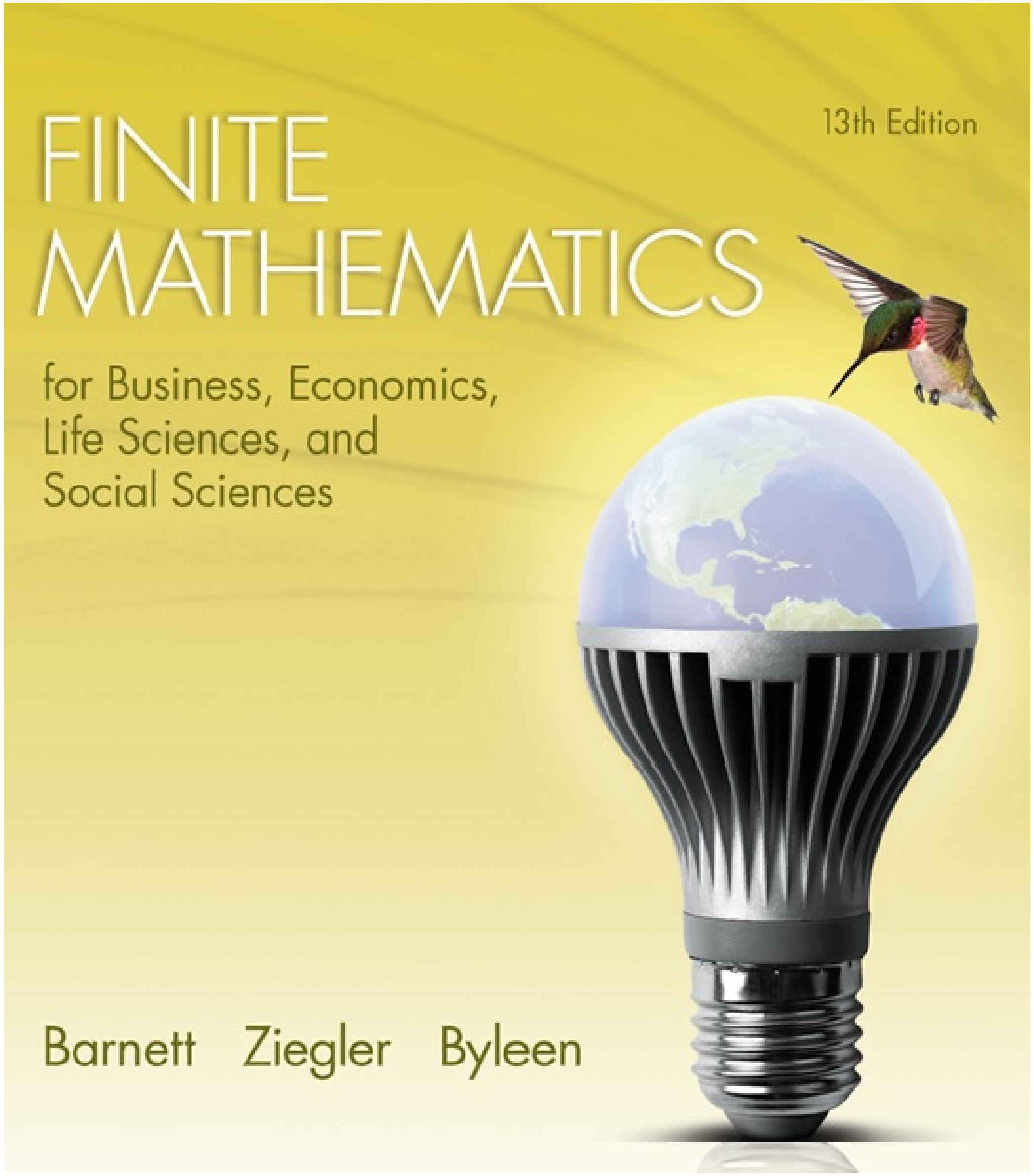


Chapter 8

Probability

Section 5

Random Variable, Probability Distribution, and Expected Value

Learning Objectives for Section 8.5

Random Variable, Probability Distribution, and Expected Value

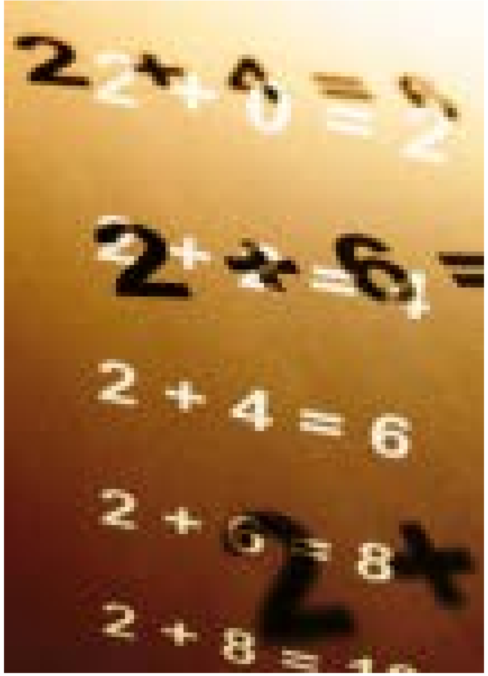
- The student will be able to identify what is meant by a random variable.
- The student will be able to create and use a probability distribution for a random variable.
- The student will be able to compute the expected value of a random variable.
- The student will be able to use the expected value of a random variable in decision-making.

Random Variables

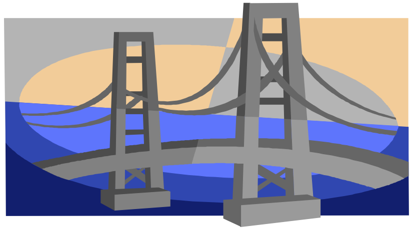
A **random variable** is a function that assigns a numerical value to each simple event in a sample space *S*.

If these numerical values are only integers (no fractions or irrational numbers), it is called a **discrete random variable**.

Note that a random variable is neither random nor a variable - it is a function with a numerical value, and it is defined on a sample space.



Examples of Random Variables



1. A function whose range is the number of speeding tickets issued on a certain stretch of I 95 S.

2. A function whose range is the number of heads which appear when 4 dimes are tossed.

3. A function whose range is the number of passes completed in a game by a quarterback.

These examples are all discrete random variables.

Probability Distributions

The simple events in a sample space S could be anything: heads or tails, marbles picked out of a bag, playing cards.

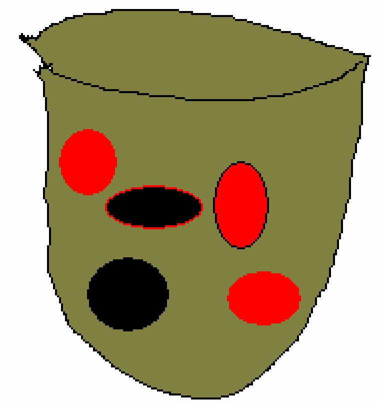
The point of introducing random variables is to associate the simple events with **numbers**, with which we can calculate.

We transfer the probability assigned to elements or subsets of the sample space to numbers. This is called the **probability distribution** of the random variable X. It is defined as

$$p(x) = P(X = x)$$

Example

- A bag contains 2 black checkers and 3 red checkers.
- Two checkers are drawn **without replacement** from this bag and the number of red checkers is noted.
- Let X = number of red checkers drawn from this bag.
- Determine the probability distribution of X and complete the table:



x	$p(x)$
0	
1	
2	

Example (continued)

- Possible values of X are 0, 1, 2. (Why?)
- $p(x = 0) = P$(black on first draw and black on second draw) =

$$P(B_1)P(B_2 \mid B_1) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

- Now, complete the rest of the table.

Hint: Find $p(x = 2)$ first, since it is easier to compute than $p(x = 1)$.

x	$p(x)$
0	1/10
1	
2	

Example (continued)

- Possible values of X are 0, 1, 2. (Why?)
- $p(x = 0) = P$(black on first draw and black on second draw) =

$$P(B_1)P(B_2 \mid B_1) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

- Now, complete the rest of the table.

Hint: Find $p(x = 2)$ first, since it is easier to compute than $p(x = 1)$.

x	$p(x)$
0	1/10
1	6/10
2	3/10

Properties of Probability Distribution

Properties: 1. $0 \leq p(x_i) \leq 1$

2. $p(x_1) + p(x_2) + \ldots + p(x_n) = 1$

The first property states that the probability distribution of a random variable *X* is a function which only takes on values between 0 and 1 (inclusive).

The second property states that the sum of all the individual probabilities must always equal one.

Example Solution

X = number of customers in line waiting for a bank teller

x	$p(x)$
0	0.07
1	0.10
2	0.18
3	0.23
4	0.32
5	0.10

- Verify that this describes a discrete random variable
- **Solution:** Variable X is discrete since its values are all whole numbers. The sum of the probabilities is one, and all probabilities are between 0 and 1 inclusive, so it satisfies the requirements for a probability distribution.

Expected Value Example

Assume X = number of heads that show when tossing three coins.

Sample space: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$$X = (0, 1, 1, 1, 2, 2, 2, 3)$$

If you perform this experiment many times and average the number of heads, you would expect to find a number close to

$$\frac{0+1+1+1+2+2+2+3}{8} = \frac{12}{8} = 1.5$$

Expected Value Example (continued)

Notice the outcomes of $x = 1$ and $x = 2$ occur three times each, while the outcomes $x = 0$ and $x = 3$ occur once each. We could calculate the average as

$$\frac{0+3\cdot 1+3\cdot 2+3}{8}=0\cdot\frac{1}{8}+1\cdot\frac{3}{8}+2\cdot\frac{3}{8}+3\cdot\frac{1}{8}$$

$$=0\cdot p(0)+1\cdot p(1)+2\cdot p(2)+3\cdot p(3)$$

Expected Value of Random Variable

The **expected value** of a random variable *X* is defined as

$$E(X) = \sum x \cdot p(x)$$

How is this interpreted?

If you perform an experiment thousands of times, record the value of the random variable every time, and average the values, you should get a number close to $E(X)$.

Computing the Expected Value

Step 1. Form the probability distribution of the random variable.

Step 2. Multiply each image value of X, x_i, by its corresponding probability of occurrence p_i ; then add the results.

Application to Business

A rock concert producer has scheduled an outdoor concert for Saturday, March 8. If it does not rain, the producer stands to make a \$20,000 profit from the concert. If it does rain, the producer will be forced to cancel the concert and will lose \$12,000 (rock star's fee, advertising costs, stadium rental, etc.)

The producer has learned from the National Weather Service that the probability of rain on March 8 is 0.4.

A) Write a probability distribution that represents the producer's profit.

B) Find and interpret the producer's "expected profit".

Application to Business Solution

(A) There are two possibilities: It rains on March 8, or it doesn't. Let *x* represent the amount of money the producer will make. So, *x* can either be \$20,000 (if it doesn't rain) or *x* = -\$12,000 (if it does rain). We can construct the following table:

	x	$p(x)$	$x*p(x)$
rain	–12,000	0.4	–4,800
no rain	20,000	0.6	12,000
			$\sum x \bullet p(x)$ =7,200

Application to Business Solution (continued)

(B) The expected value is interpreted as a long-term average. The number \$7,200 means that if the producer arranged this concert many times in identical circumstances, he would be ahead by \$7,200 per concert on the average. It does not mean he will make exactly \$7,200 on March 8. He will either lose \$12,000 or gain \$20,000.