Review 2 Solution

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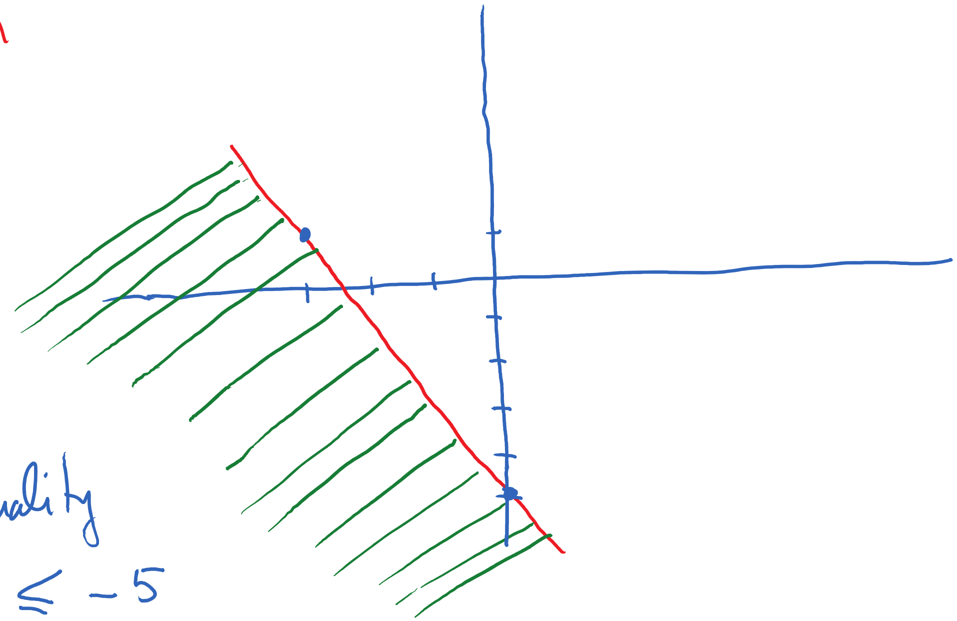
① C

x	y
0	-5
-3	1



Plug (0,0) into inequality

$2x + y \leq -5$

$0 < -5$ (Not true)

② through ④. Similar idea. Graph the lines. Select test points to choose the correct region for the inequalities.

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⑤

Corner points	$P = x + y$
$(0,0)$	0
$(0,12)$	12
$(8,4)$	12
$(9,0)$	9

So, Max $P = 12$ at $(0,12)$ or $(8,4)$.

⑥ First graph the feasible region for the inequalities $x + 2y \geq 10$ and $3x + y \geq 10$

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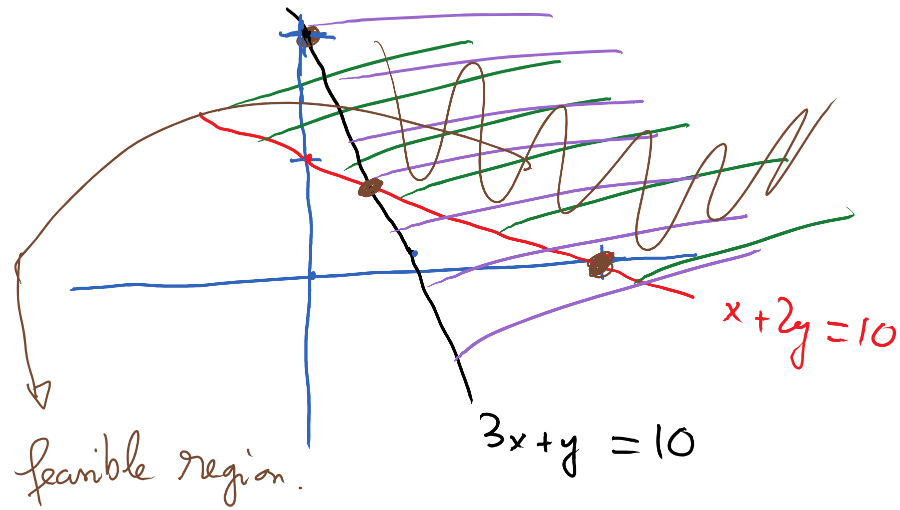
x	y
0	5
10	0

$x + 2y \geq 10$

Test point $(0,0)$

x	y
0	10
3	1

$3x + y \geq 10$



Feasible region is unbounded: corner points are $(2,4)$; $(10,0)$ and $(0,10)$.

Objective function $z = 2x + 4y$ has min value of 20 at $(2,4)$ and $(10,0)$ and any point on the line $x+2y=10$ in between those 2 points.

(7) Introduce slack variables:

$$
\begin{aligned}
2x_1 + 5x_2 + s_1 \quad &= 9 \\
3x_1 + 3x_2 \quad + s_2 &= 2 \\
-4x_1 - x_2 \quad + P &= 0
\end{aligned}
$$

$$
\begin{array}{c}
 \\ s_1 \\ s_2 \\ P
\end{array}
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & s_1 & s_2 & P & \end{array} \\
\left(\begin{array}{ccccc|c}
2 & 5 & 1 & 0 & 0 & 9 \\
3 & 3 & 0 & 1 & 0 & 2 \\
-4 & -1 & 0 & 0 & 1 & 0
\end{array}\right)
\end{array}
$$

(8) Introduce slack variables:

$$
\begin{aligned}
3x_1 + 8x_2 + s_1 \quad &= 5 \\
3x_1 + 5x_2 \quad + s_2 \quad &= 6 \\
-8x_1 + 6x_2 \quad + s_3 \quad &= 32 \\
6x_2 \quad + s_4 &= 3 \\
-5x_1 - 3x_2 \quad + P &= 0
\end{aligned}
$$

exit

enter

$$
\begin{array}{c|ccccccc|c}
 & x_1 & x_2 & s_1 & s_2 & s_3 & s_4 & P & \\
\hline
s_1 & 3 & 8 & 1 & 0 & 0 & 0 & 0 & 5 \\
s_2 & \boxed{3} & 5 & 0 & 1 & 0 & 0 & 0 & 6 \\
s_3 & -8 & 6 & 0 & 0 & 1 & 0 & 0 & 32 \\
s_4 & 0 & 6 & 0 & 0 & 0 & 1 & 0 & 3 \\
P & -5 & -3 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

pivot

pivot row

pivot column

9

$$2x_1 + 5x_2 + s_1 = 19$$

$$3x_1 + 3x_2 + s_2 = 18$$

$$-4x_1 - x_2 + z = 0$$

$$\begin{array}{c|c}
 & \begin{array}{ccccc|c} \overset{\text{enter}}{x_1} & x_2 & s_1 & s_2 & z & \end{array} \\
\begin{array}{c} s_1 \\ {}^{\text{exit}} s_2 \\ z \end{array} & \left(\begin{array}{ccccc|c} 2 & 5 & 1 & 0 & 0 & 19 \\ \boxed{3} & 3 & 0 & 1 & 0 & 18 \\ -4 & -1 & 0 & 0 & 1 & 0 \end{array}\right)
\end{array}$$

pivot

$\rightarrow$ pivot row

$\downarrow$ pivot column

10

$$
\begin{array}{c} \\ s_1 \\ s_2 \\ z \end{array}
\begin{array}{c}
\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & \end{array} \\
\left(\begin{array}{cccccc|c}
2 & 1 & 4 & 1 & 0 & 0 & 48 \\
2 & 4 & 1 & 0 & 1 & 0 & 32 \\
-1 & -3 & -2 & 0 & 0 & 1 & 0
\end{array}\right)
\end{array}
\xrightarrow{\frac{1}{2}R_1 \to R_1}
$$

$R_2 - 2R_1 \to R_1$

$R_3 + R_1 \to R_1$

Note: the problem asked us to pivot about the circled element. It doesn't mean the element is the true pivot element in the tableau.

$$
\begin{array}{c} \\ x_1 \\ s_2 \\ z \end{array}
\begin{array}{c}
\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & \end{array} \\
\left(\begin{array}{cccccc|c}
1 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 24 \\
0 & 3 & -3 & -1 & 1 & 0 & -16 \\
0 & -\frac{5}{2} & 0 & \frac{1}{2} & 0 & 1 & 24
\end{array}\right)
\end{array}
$$

So, after pivoting once around the circled element

$x_1 = 24$; $s_2 = -16$, $z = 24$

$x_2, x_3, s_1 = 0$

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(11)

$$x_1 + 5x_2 + 7x_3 + s_1 = 8$$

$$x_1 + 4x_2 + 11x_3 + s_2 = 9$$

$$-7x_1 - 2x_2 - x_3 + P = 0$$

enter

exit

$$\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & s_1 & s_2 & P \\
s_1 & \left(\begin{array}{cccccc|c} \boxed{1} & 5 & 7 & 1 & 0 & 0 & 8 \\ 1 & 4 & 11 & 0 & 1 & 0 & 9 \\ -7 & -2 & -1 & 0 & 0 & 1 & 0 \end{array}\right) & & & & & \\
s_2 & & & & & & \\
P & & & & & &
\end{array}$$

pivot

pivot row

pivot column

$$\xrightarrow[R_3 + 7R_1]{R_2 - R_1}$$

$$\begin{array}{c|c}
 & x_1 \quad x_2 \quad x_3 \quad s_1 \quad s_2 \quad P \\
\begin{array}{c} x_1 \\ s_2 \\ P \end{array} & \left(\begin{array}{cccccc|c} 1 & 5 & 7 & 1 & 0 & 0 & 8 \\ 0 & -1 & 4 & -1 & 1 & 0 & 1 \\ 0 & 33 & 48 & 7 & 0 & 1 & 56 \end{array}\right)
\end{array}$$

So, max $P = 56$

when $x_1 = 8$

$x_2 = x_3 = 0$

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(12) Transpose of $\begin{pmatrix} 1 & 6 & 9 & 3 \\ 1 & 7 & 8 & 6 \end{pmatrix}$ is $\begin{pmatrix} 1 & 1 \\ 6 & 7 \\ 9 & 8 \\ 3 & 6 \end{pmatrix}$

(13) Original matrix:

$$A = \begin{pmatrix} 2 & 3 & 60 \\ 1 & 4 & 40 \\ 3 & 1 & 1 \end{pmatrix}$$

Transpose: $\begin{pmatrix} 2 & 1 & 3 \\ 3 & 4 & 1 \\ 60 & 40 & 1 \end{pmatrix}$

Dual problem

Maximize

$$P = 60y_1 + 40y_2$$

subject to

$$2y_1 + y_2 \leq 3$$

$$3y_1 + 4y_2 \leq 1$$

14 Original Matrix

$$A = \begin{pmatrix} 8 & 1 & 0 & 2 \\ 0 & 8 & 5 & 16 \\ 6 & 1 & 5 & 1 \end{pmatrix}$$

Transpose

$$A^T = \begin{pmatrix} 8 & 0 & 6 \\ 1 & 8 & 1 \\ 0 & 5 & 5 \\ 2 & 16 & 1 \end{pmatrix}$$

Dual problem:

Maximize $\underline{P} = 2y_1 + 16y_2$ subject to

$8y_1 \leq 6$; $y_1 + 8y_2 \leq 1$; $5y_2 \leq 5$, $y_1, y_2 \geq 0$

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$$A = \begin{pmatrix} 2 & 3 & 9 \\ 2 & 1 & 11 \\ 5 & 3 & 1 \end{pmatrix} ; \quad A^T = \begin{pmatrix} 2 & 2 & 5 \\ 3 & 1 & 3 \\ 9 & 11 & 1 \end{pmatrix}$$

Dual problem: Maximize $P = 9y_1 + 11y_2$

Subject to $2y_1 + 2y_2 \leq 5$; $3y_1 + y_2 \leq 3$

$$2y_1 + 2y_2 + x_1 = 5$$

$$3y_1 + y_2 + x_2 = 3$$

$$-9y_1 - 11y_2 + P = 0$$

$$\begin{array}{c|c} & \begin{array}{cccccc} y_1 & \overset{\text{enter}}{y_2} & x_1 & x_2 & P & \end{array} \\ \text{exit } x_1 & \\ x_2 & \\ P & \end{array} \left(\begin{array}{ccccc|c} 2 & \boxed{2} & 1 & 0 & 0 & 5 \\ 3 & 1 & 0 & 1 & 0 & 3 \\ -9 & -11 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[R_2 - R_1 \to R_2 \;;\; R_3 + 11R_1 \to R_3]{\frac{1}{2}R_1 \to R_1}$$

$$\begin{array}{c} \\ y_2 \\ x_2 \\ P \end{array}
\begin{array}{c}
\begin{array}{cccccc} y_1 & y_2 & x_1 & x_2 & P & \end{array} \\
\begin{pmatrix}
1 & 1 & \frac{1}{2} & 0 & 0 & \frac{5}{2} \\
2 & 0 & -\frac{1}{2} & 1 & 0 & \frac{1}{2} \\
2 & 0 & \frac{11}{2} & 0 & 1 & \frac{55}{2}
\end{pmatrix}
\end{array}$$

So, $\min f = \frac{55}{2}$ when $x = \frac{11}{2}$; $y = 0$

(Recall: read bottom row for this type of problem

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(16) # couselors $= x_1$

aides $= x_2$.

Minimize cost $C = 2400x_1 + 1100x_2$.

Subject to: $x_1 + x_2 \geq 30$

$x_2 \geq 10$

$3x_1 - 2x_2 \geq 0$

$$A = \begin{pmatrix} 1 & 1 & 30 \\ 0 & 1 & 10 \\ 3 & -2 & 0 \\ 2400 & 1100 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 0 & 3 & 2400 \\ 1 & 1 & -2 & 1100 \\ 30 & 10 & 0 & 1 \end{pmatrix}$$

Dual problem: Maximize $P = 30y_1 + 10y_2$

Subject to : $y_1 + 3y_3 \leq 2400$

$y_1 + y_2 - 2y_3 \leq 1100$

$$y_1 + 3y_3 + x_1 = 2400$$

$$y_1 + y_2 - 2y_3 + x_2 = 1100$$

$$-30y_1 - 10y_2 + P = 0$$

$$\begin{array}{r|c}
 & \begin{array}{cccccc|c} \overset{\text{enter}}{y_1} & y_2 & y_3 & x_1 & x_2 & P & \end{array} \\
\begin{array}{r} x_1 \\ \text{exit } x_2 \\ P \end{array} & \left(\begin{array}{cccccc|c} 1 & 0 & 3 & 1 & 0 & 1 & 2400 \\ \boxed{1} & 1 & -2 & 0 & 1 & 0 & 1100 \\ -30 & -10 & 0 & 0 & 0 & 1 & 0 \end{array}\right)
\end{array}$$

$$\xrightarrow[R_3 + 30R_2 \to R_3]{R_1 - R_2 \to R_1}$$

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$$
\begin{array}{c} \text{exit} \\ x_1 \\ y_1 \\ P \end{array}
\begin{array}{c}
\begin{array}{cccccc|c} & & \text{enter} & & & & \\ y_1 & y_2 & y_3 & x_1 & x_2 & P & \end{array} \\
\left(\begin{array}{cccccc|c}
0 & -1 & \boxed{5} & 1 & -1 & 1 & 1300 \\
1 & 1 & -2 & 0 & 1 & 0 & 1100 \\
0 & 20 & -60 & 0 & 30 & 1 & 33000
\end{array}\right)
\end{array}
\xrightarrow[\substack{R_2 + 2R_1 \to R_2 \\ R_3 + 60R_1 \to R_3}]{\frac{1}{5}R_1 \to R_1}
$$

$$
\begin{array}{c} \\ y_3 \\ y_1 \\ P \end{array}
\begin{array}{c}
\begin{array}{cccccc|c} y_1 & y_2 & y_3 & x_1 & x_2 & P & \end{array} \\
\left(\begin{array}{cccccc|c}
0 & -0.2 & 1 & 0.2 & -0.2 & 0.2 & 260 \\
1 & 0.6 & 0 & 0.4 & 0.6 & 0.4 & 1620 \\
0 & 8 & 0 & 12 & 18 & 13 & 48600
\end{array}\right)
\end{array}
$$

So, $\min C = 48600$ when $x_1 = 12$; $x_2 = 18$

(12 counselors, 18 aides)