Anti Derivative.

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Definition of an antiderivative of a function.

f : function defined on an interval I.

A function F is an antiderivative of f on I if

$$F'(x) = f(x) \text{ for every } x \text{ in } I.$$

E.g: $f(x) = x$. defined on $(-\infty, \infty)$

Find an antiderivative for f.

$F(x) = \frac{x^2}{2}$. $F'(x) = x = f(x)$

$F(x) = \frac{x^2}{2}$ is an antiderivative of $f(x) = x$

$G(x) = \frac{x^2}{2} + 10$. $G'(x) = f(x)$

$H(x) = \frac{x^2}{2} + \pi$; $H'(x) = f(x)$

Any function of the form $\boxed{F(x) = \frac{x^2}{2} + C}$, where C is a constant is an antiderivative of $f(x) = x$

Theorem: If $F(x)$ and $G(x)$ are 2 different antiderivatives of $f(x)$, then $F(x) = G(x) + C$.

Therefore, if $F(x)$ is an antiderivative of $f(x)$, then the most general antiderivative of $f(x)$ has the form

$$F(x) + C \text{ ; } C : \text{constant}.$$

E.g. Find the general antiderivative of the given function.

① $f(x) = x^2$.

$$F(x) = \frac{x^3}{3} + C$$

② $f(x) = x^3$

$$F(x) = \frac{x^4}{4} + C$$

③ $f(x) = x^{2017}$

$$F(x) = \frac{x^{2018}}{2018} + C.$$

In general. $f(x) = x^n$. $F(x) = \frac{x^{n+1}}{n+1} + C$.

The most general antiderivative for $f(x) = x^n$ where n is any real number; $n \neq -1$ is:

$$F(x) = \frac{x^{n+1}}{n+1} + C$$

Consider the case where $n = -1$.

$$f(x) = x^{-1} = \frac{1}{x}$$

$$F(x) = \ln|x| + C$$

$f(x) = \cos x$, $F(x) = \sin x + C$

$f(x) = \sin x$, $F(x) = -\cos x + C$

Important Notation: notation for indefinite integral.

$$\boxed{\int}\boxed{f(x)}\boxed{dx} := \text{the most general antiderivative of } f(x)$$

integrand

variable of integration.

integrand

- variable of integration.

integral sign

$$\int x\,dx = \frac{x^2}{2} + C. \qquad \int \sin x\,dx = -\cos x + C.$$

Table of useful indefinite integrals.

Function	Indefinite integral
$f(x) = x^n$; $n \neq -1$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$; $n \neq -1$
$f(x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C.$
$f(x) = k$, k: constant	$\int k\,dx = kx + C$
$f(x) = e^x$	$\int e^x\,dx = e^x + C$
$f(x) = \sin x$	$\int \sin x\,dx = -\cos x + C$
$f(x) = \cos x$	$\int \cos x\,dx = \sin x + C.$
$f(x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C.$
$f(x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$f(x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan x + C$
$f(x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C.$

Useful properties of indefinite integrals.

Useful properties of indefinite integrals

$$\int \left(f(x) \pm g(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Note: We DO NOT have:

$$\cancel{\int f(x)\cdot g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx}$$

$$\cancel{\int \frac{f(x)}{g(x)}\,dx = \frac{\int f(x)\,dx}{\int g(x)\,dx}}$$

$$\int c\cdot f(x)\,dx = c\int f(x)\,dx$$

$$\int 2017\cdot e^x\,dx = 2017\int e^x\,dx = 2017e^x + C.$$

E.x. Find the given indefinite integral

1. $\int \left(7x^{2/5} + 8x^{-4/5}\right) dx$
2. $\int \left(2\sin x - \sec^2 x\right) dx$
3. $\int e^2\,dx = e^2\cdot x + C$
4. $\int \left(4\sqrt{x} - \sqrt[4]{x}\right) dx$
5. $\int (x+1)(2x-1)\,dx$
6. $\int \frac{2x^5 - \sqrt{x}}{x}\,dx$

* ⑦ $\int \frac{2+x^2}{1+x^2}\,dx$

Solved in class.

Basic Initial Value Problems.

E.g. Find the function $f(x)$ given that

$$f'(x) = \frac{2}{x^2} - \frac{x^2}{2}.$$

$$f(1) = 0$$

Sol: $f(x) = \int \left(\frac{2}{x^2} - \frac{x^2}{2}\right) dx = \int \left(2\cdot x^{-2} - \frac{x^2}{2}\right) dx = 2\cdot\frac{x^{-1}}{-1} - \frac{x^3}{6} + C$

$$f(x) = -\frac{2}{x} - \frac{x^3}{6} + C$$

$f(1) = 0 \longrightarrow -\frac{2}{1} - \frac{1}{6} + C = 0$

$$-2 - \frac{1}{6} + C = 0$$

$$-\frac{13}{6} + C = 0 \rightarrow C = \frac{13}{6}.$$

$$f(x) = -\frac{2}{x} - \frac{x^3}{6} + \frac{13}{6}.$$

E.g. Find $f(x)$ given:

$$f''(x) = 2e^x + 3\sin x$$

$$\boxed{f(0) = 0 \quad ; \quad f(\pi) = 0}$$

$f'(x) = \int (2e^x + 3\sin x)\,dx = 2e^x - 3\cos x + C_1.$

$f(x) = \int (2e^x - 3\cos x + C_1)\,dx$

$$f(x) = \int \left(2e^x - 3\cos x + C_1\right) dx$$

$$f(x) = 2e^x - 3\sin x + C_1 x + C_2$$

$$f(0) = \underbrace{2e^0 - 3\sin 0 + C_1 \cdot 0 + C_2} = 0$$

$$2 + C_2 = 0 \longrightarrow C_2 = -2.$$

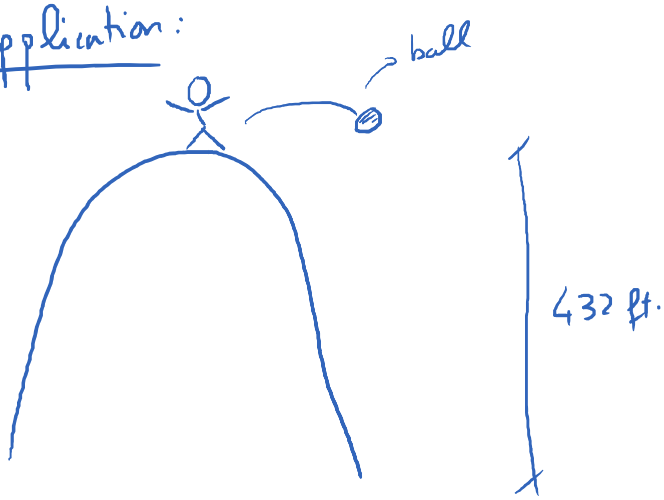
$$f(\pi) = 2e^\pi - 3\sin(\pi) + C_1 \cdot \pi - 2 = 0$$

$$2e^\pi + C_1 \cdot \pi - 2 = 0$$

$$C_1 \cdot \pi = 2 - 2e^\pi. \quad C_1 = \frac{2 - 2e^\pi}{\pi}.$$

$$f(x) = 2e^x - 3\sin x + \frac{2 - 2e^\pi}{\pi} \cdot x - 2.$$

<u>An application:</u>



A ball is thrown upward with an initial speed $\underline{v_0 = 48 \text{ ft/s}}$ from the top of a cliff 432 ft above ground.

Find the position function $s(t)$ of ball.

Constant acceleration due to gravity is -32 ft/s^2.

$$a(t) = -32$$

$$v(t) = \int -32\, dt = -32t + C$$

Since $v_0 = 48 \longrightarrow C = 48$

$$v(t) = -32t + 48.$$

$$s(t) = \int (-32t + 48)\, dt = -32 \cdot \frac{t^2}{2} + 48t + \boxed{C}$$

$$s(t) = \int(-32t+48)\,dt = -32\cdot\frac{t^2}{2} + 48t + \boxed{C}$$

$$= \boxed{-16t^2 + 48t + 432}$$

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E.g.