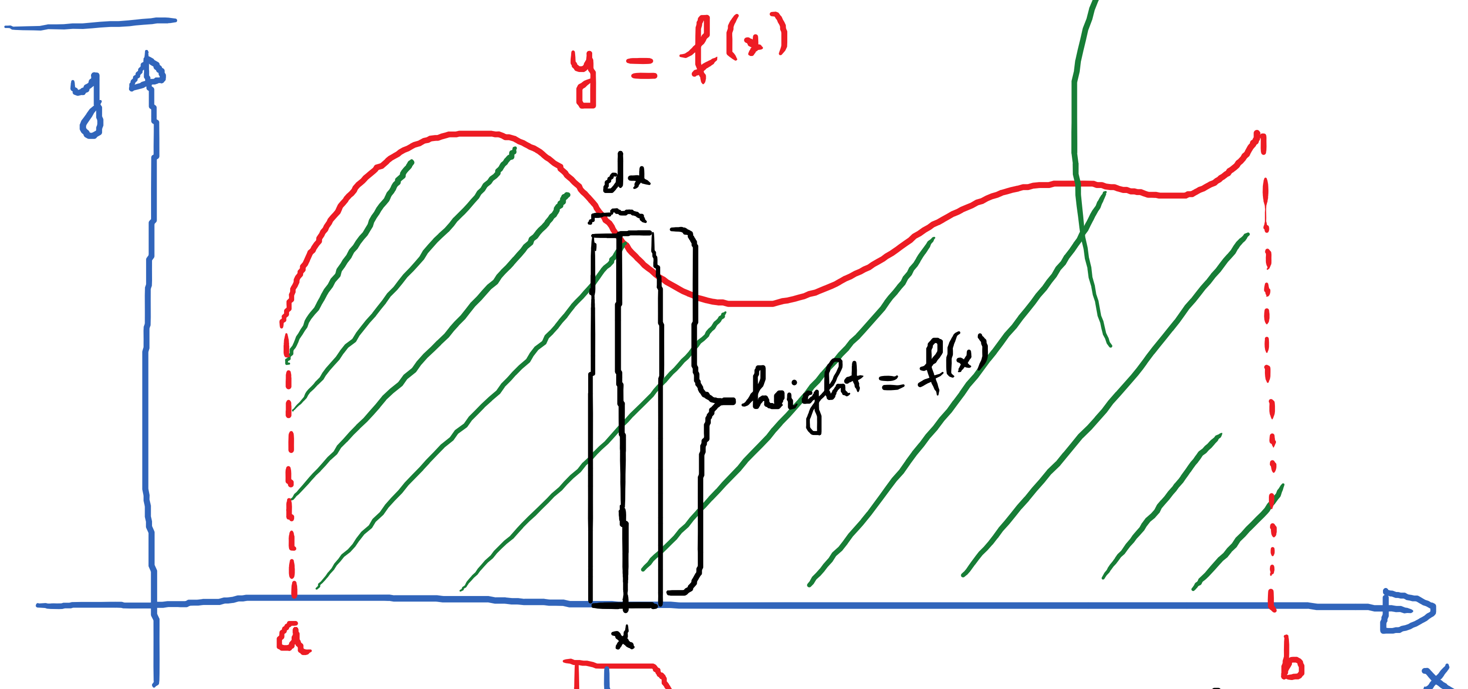
Areas Between Curves

Wednesday, August 16, 2017 7:49 AM

Area under the curve on [a, b]

Recall:



$$\text{Area} = \boxed{\int_a^b} \boxed{\underbrace{f(x)}_{\text{height of small rectangle}}\ \underbrace{dx}_{\text{width of small rectangle}}}$$

area of a small rectangle

Infinite Sum

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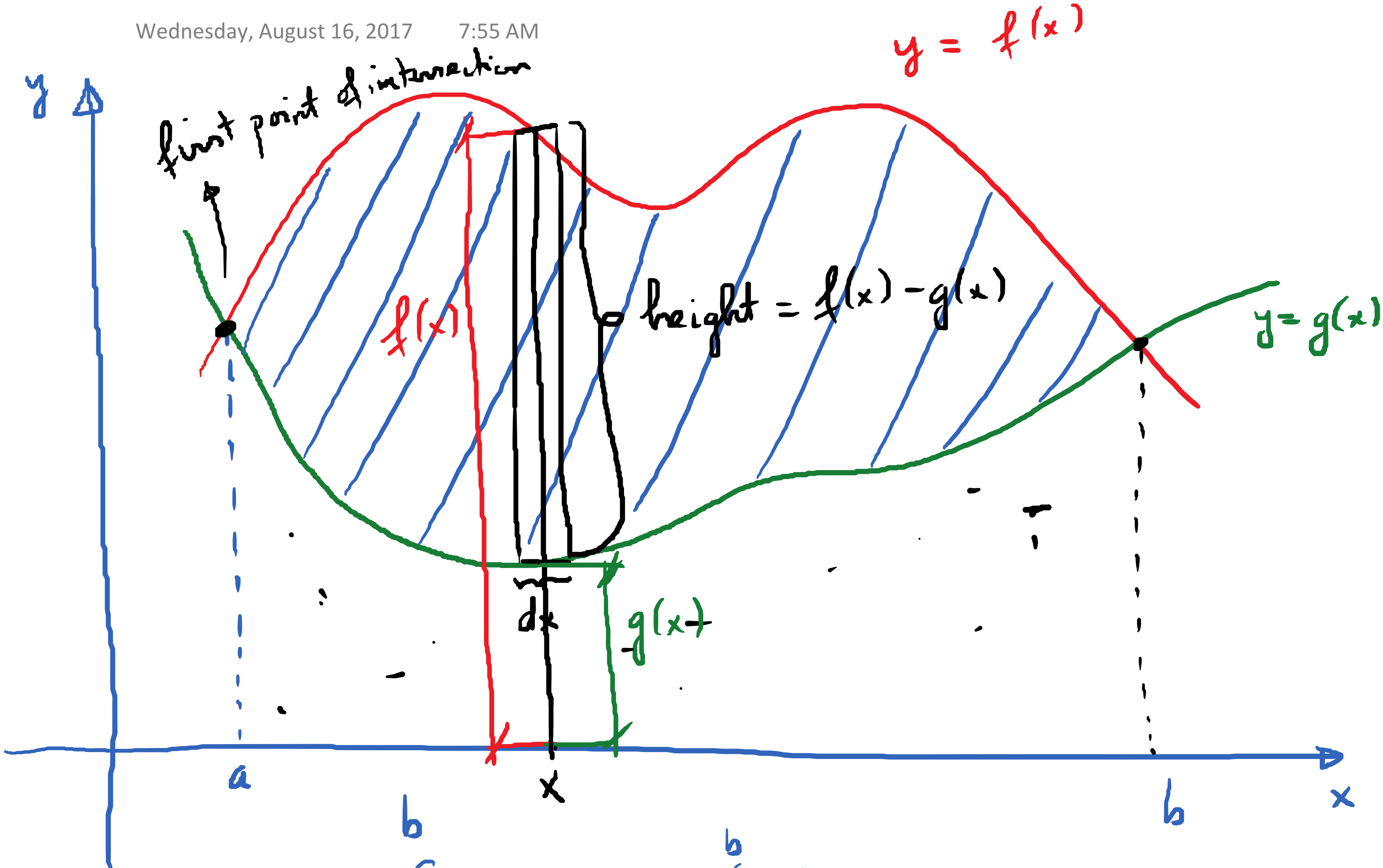
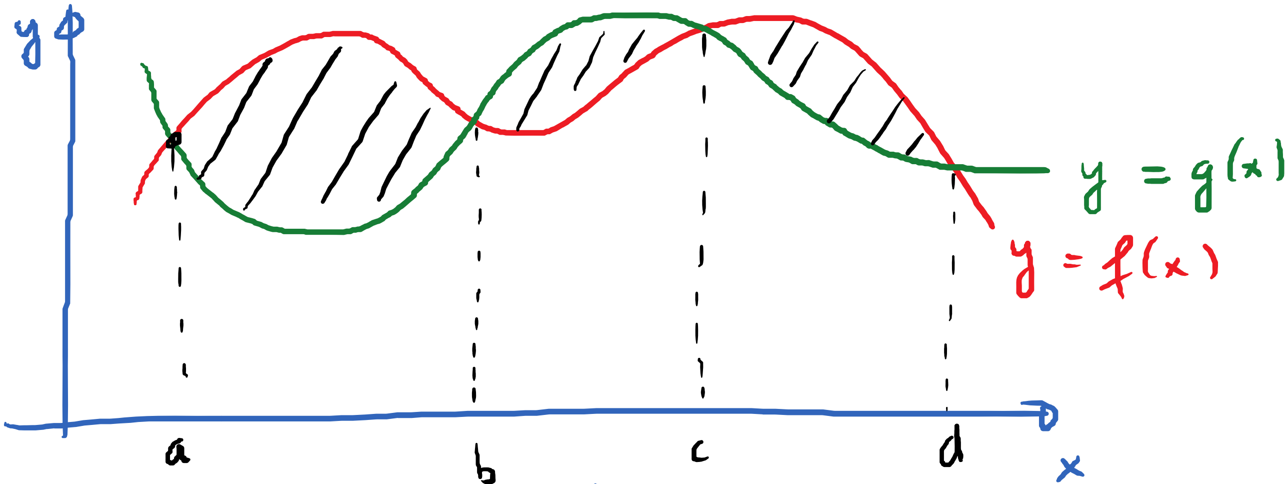


$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

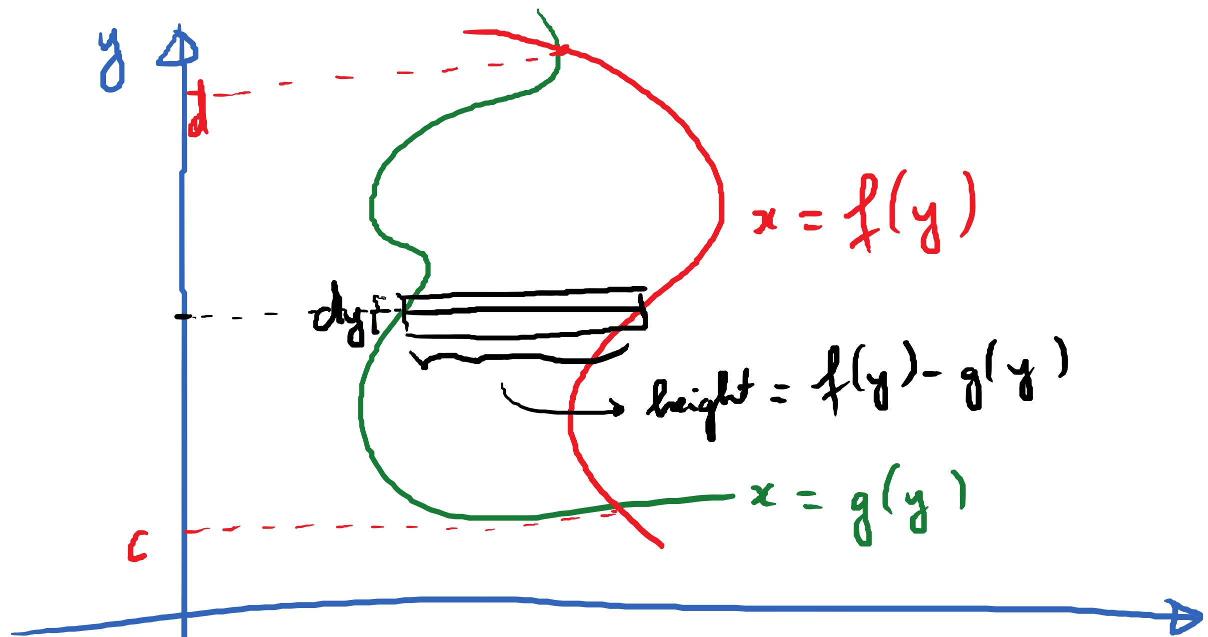
$$\text{Area} = \underbrace{\boxed{\int_a^b}}_{\text{Sum}} \overbrace{\underbrace{\big(f(x) - g(x)\big)}_{\text{height}}\,\underbrace{dx}_{\text{width}}}^{\text{small area}}$$

$$\text{Area} = \int_a^b (\text{top curve} - \text{bottom curve})\,dx$$

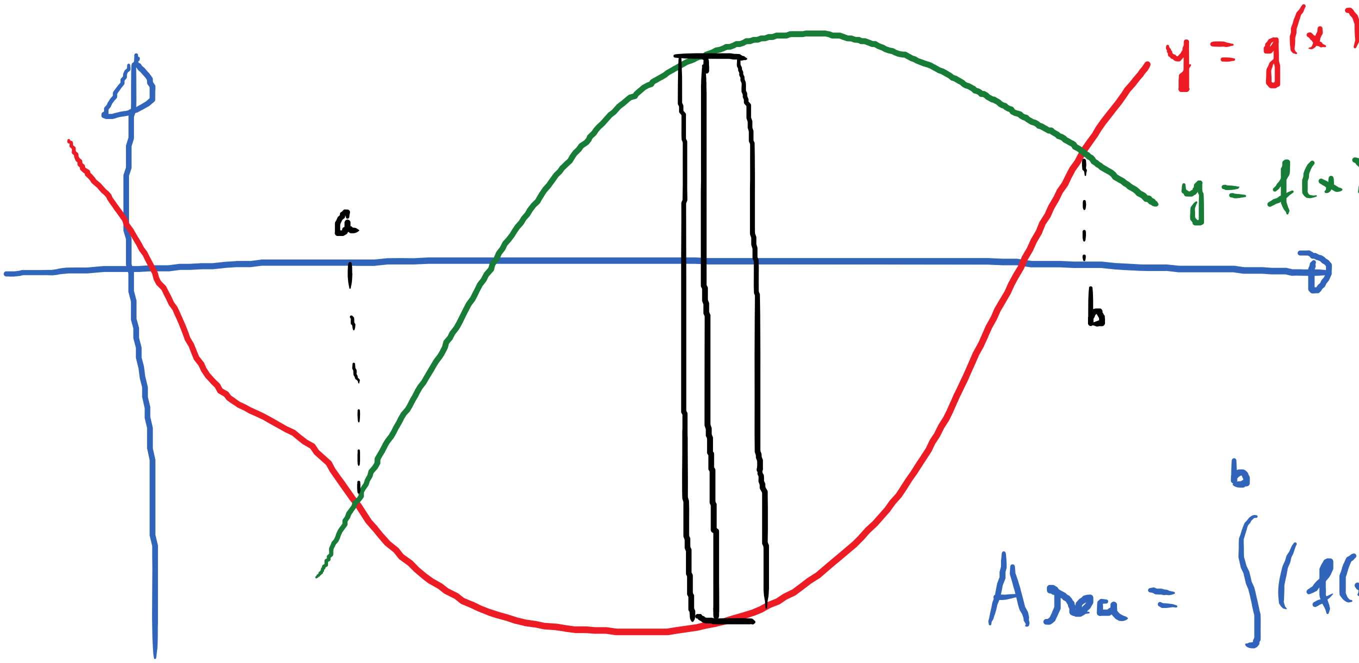


Shaded area = $\int_a^b (f(x) - g(x))\,dx + \int_b^c (g(x) - f(x))\,dx$

$+ \int_c^d (f(x) - g(x))\,dx$



$$\text{Area} = \int_c^d \Big(\underbrace{f(y)}_{\text{right most curve}} - \underbrace{g(y)}_{\text{left most curve}} \Big)\, dy$$



$$\text{Area} = \int_a^b (f(x) - g(x))\, dx$$

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HW # 1

$$\int_{?}^{?} \Big[\underbrace{(x^3-2x^2+2)}_{\text{top}} - \underbrace{(3x^2+3x-13)}_{\text{bottom}}\Big]\,dx$$

Find points of Intersection : Set the 2 equations equal to each other

$$x^3-2x^2+2 = 3x^2+3x-13$$

$$\underbrace{x^3-5x^2}\underbrace{-3x+15} = 0$$

$$x^2(x-5)-3(x-5) = 0$$

$$(x^2-3)\cdot(x-5) = 0$$

$$x^2-3=0 \quad ; \quad x-5=0$$

$$x = \pm\sqrt{3} \quad ; \quad x = 5$$

$$\int_{-\sqrt{3}}^{\sqrt{3}} (x^3-5x^2-3x+15)\,dx = \cdots$$

11:

$$\int_{? \to -1}^{? \to 9/2} \left[(7y - y^2) - (-9 + y^2)\right] dy$$

Find points of Intersection:

$$7y - y^2 = -9 + y^2$$

$$-2y^2 + 7y + 9 = 0$$

Trick: * Find ac product: $(-2)\cdot 9 = -18$

* Factor it in a way that sum of 2 factors gives you the middle term $\longrightarrow 18 = (-2)\cdot 9$

$-2 + 9 = 7$

$$\underbrace{-2y^2 - 2y} + 9y + 9 = 0$$

$$-2y(y+1) + 9(y+1) = 0$$

$$(-2y + 9)(y + 1) = 0$$

$$y = 9/2 \quad ; \quad y = -1$$

HW 12

$$x = \frac{-y^2+2}{2}$$

$$x = |y|$$

$$\int_?^? \left(\frac{-y^2+2}{2} - |y|\right) dy$$

$$|y| = \frac{-y^2+2}{2}.$$ Consider $y > 0$

$|y| = y$.

$$y = \frac{-y^2+2}{2}. \qquad 2y = -y^2+2$$

$$y^2 + 2y - 2 = 0$$

$$y = \frac{-2 \pm \sqrt{4 - 4\cdot 1\cdot(-2)}}{2} = \frac{-2 \pm \sqrt{12}}{2}$$

Choose $y = \dfrac{-2+\sqrt{12}}{2}$

$$\text{Area} = 2 \cdot \int_0^{\frac{-2+\sqrt{12}}{2}} \left(\frac{-y^2+2}{2} - y \right) dy$$

$$= \cdots$$