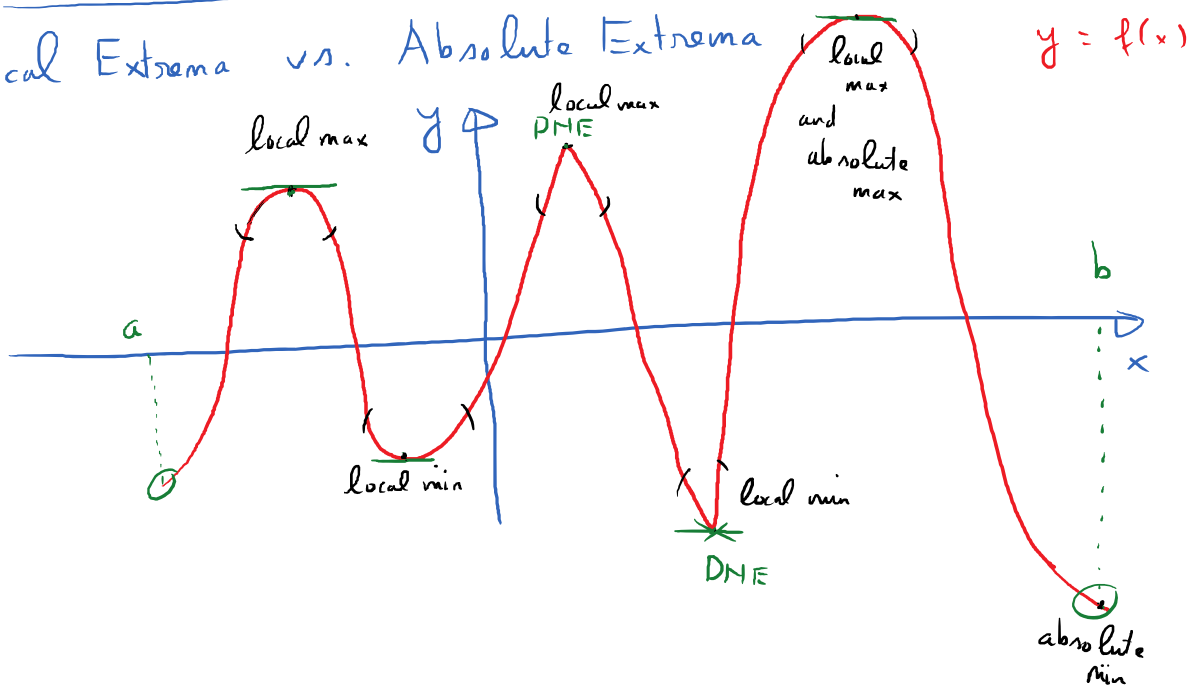
4.3. Find Maxima and Minima of a Function

Tuesday, August 1, 2017 7:31 AM

Goal: (1) Find critical numbers of a function.

(2) Use the closed interval method to find maxima and minima of a function f over an interval $[a, b]$.

Local Extrema vs. Absolute Extrema



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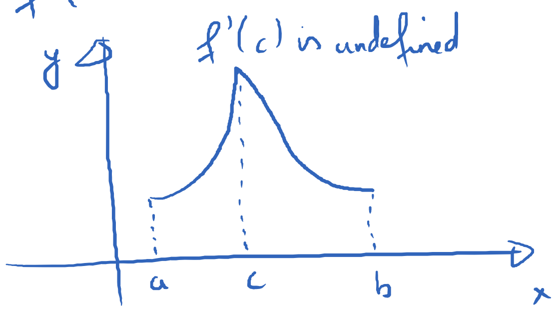
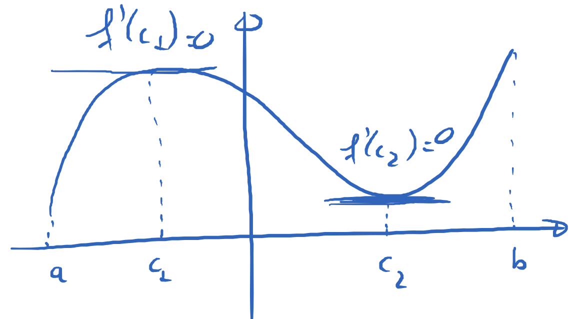
Critical Points (Critical Numbers) of a function

<u>Def of a critical number</u>: let c be a number in the domain of a function f, c is called a critical number of f if $f'(c) = 0$ or $f'(c)$ is undefined



Derivative undefined



Derivative = 0

To find critical #'s of f. First find $f'(x)$.

Second, find values in domain of f s.t. $\begin{cases} f'(x) = 0 \\ f'(x) \text{ is undefined} \end{cases}$

<u>E.g.</u> $f(x) = x^3 - 6x^2 + 9x + 1$. Find critical #'s of f.

Domain: $(-\infty, \infty)$

$f'(x) = 3x^2 - 12x + 9$.

$f'(x)$ — undefined: None

$f'(x)$ — $f'(x) = 0$

$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$\boxed{x = 1 \; ; \; x = 3}$

critical #'s.

Ex. Find critical points of the given function

(1) $f(x) = \dfrac{4x}{1+x^2}$ (2) $h(x) = x^{\frac{3}{5}}(4-x)$.

(3) Can you draw the graph of a function that has no critical points.

Solved in class.

E.g. $f(x) = x \cdot e^{-x^2/8}$. Critical #'s

Domain: $(-\infty, \infty)$

$$f'(x) = 1 \cdot e^{-x^2/8} + x \cdot e^{-x^2/8} \cdot \left(-\frac{1}{4}x\right)$$

$$= e^{-x^2/8} - \frac{1}{4}x^2 \cdot e^{-x^2/8}$$

$$f'(x) = e^{-x^2/8}\left(1 - \frac{1}{4}x^2\right)$$

undefined: None

$= 0$

$f'(x) = 0$. $\boxed{\cancel{e}^{-x^2/8}} \cdot \boxed{\left(1 - \frac{1}{4}x^2\right)} = 0$

$\boxed{e^{-x^2/8} = 0} \longrightarrow$ No solution

$1 - \frac{1}{4}x^2 = 0$

$-\frac{1}{4}x^2 = -1$

$x^2 = 4$

$x = \pm 2$

Critical #'s: $\boxed{x = 2;\ x = -2}$

E.g. $g(x) = 4\sqrt{x} - x^2$. Find Critical #'s.

Domain: $[0, \infty)$

$$g'(x) = 4 \cdot \frac{1}{2\sqrt{x}} - 2x = \frac{2}{\sqrt{x}} - 2x$$

undefined

$g'(x) = 0$

$g'(x)$ is undefined when $\boxed{x = 0}$ ← critical pt.

$g'(x) = 0$; $\frac{2}{\sqrt{x}} - 2x = 0$

$$\frac{2}{\sqrt{x}} = 2x$$

$$\frac{1}{\sqrt{x}} = x$$

$$1 = x\sqrt{x}$$

$$1 = x^2 \cdot x$$

$$1 = x^3$$

$$\boxed{x = 1}$$ ← critical point.

Closed Interval Method to find absolute max/min of a function f on an closed interval $[a, b]$.

1. Find all the critical points of f within $[a, b]$.
2. Evaluate f at these critical points in 1
3. Evaluate f at the endpoints $x = a$ and $x = b$.
4. The largest value in 2 and 3 is the abs. max

   The smallest value in 2 and 3 is the abs. min

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 5]$

Find abs. max / min of f on $[0, 5]$

① Before critical points $x = 1$; $x = 3$

② Find $f(1)$; $f(3)$

$$f(1) = (1)^3 - 6(1)^2 + 9(1) + 1 = \boxed{5}$$

$$f(3) = (3)^3 - 6(3)^2 + 9(3) + 1 = \boxed{1}$$

③ Find $f(0)$; $f(5)$

$f(0) = \boxed{1}$; $f(5) = (5)^3 - 6(5)^2 + 9(5) + 1 = \boxed{21}$

④ Conclusion : abs max = 21 when $x = 5$.

Pt $\boxed{(5, 21)}$

abs min = 1 when $x = 0$; $x = 3$.

$\boxed{(0, 1) ; (3, 1)}$