# 3.1 - Definition of the Derivative.

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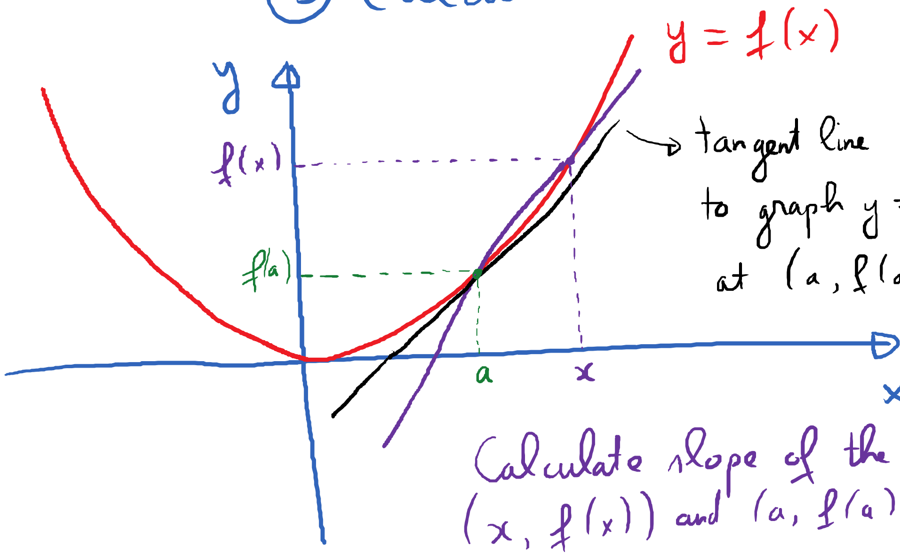
Goals: (1) Solve the tangent line problem

(2) Definition of the derivative of a function at a given point

(3) Calculate the derivative using definition.




tangent line to graph $y = f(x)$ at $(a, f(a))$. Equation? Slope?

Call the slope $m_{tangent}$.

Calculate slope of the secant line connecting $(x, f(x))$ and $(a, f(a))$

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$$\text{Slope} = \frac{f(x) - f(a)}{x - a}$$

So, $$\boxed{m_{\text{tangent}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$ provided that the limit exists.

E.g. $f(x) = x^2$. (a) Find the slope of the tangent line to the graph of $f$ at the point $a = 3$; $f(a) = 9$. Point $(3, 9)$.

(b) Write the point slope equation of the tangent line at $(3,9)$.

(a) $$m_{\text{tangent}} = \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} (x+3) = \boxed{6}$$

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(b) Point - Slope equation:

$$y - y_1 = m(x - x_1)$$

$m$ = slope = 6

$$y - 9 = 6(x - 3)$$

Slope - intercept: $y - 9 = 6x - 18$

$$\boxed{y = 6x - 9}$$

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E.g. $f(x) = \frac{3}{x}$.

Find the slope and the equation of the tangent line to the graph of $f$ at $(3, 1)$

Solved!

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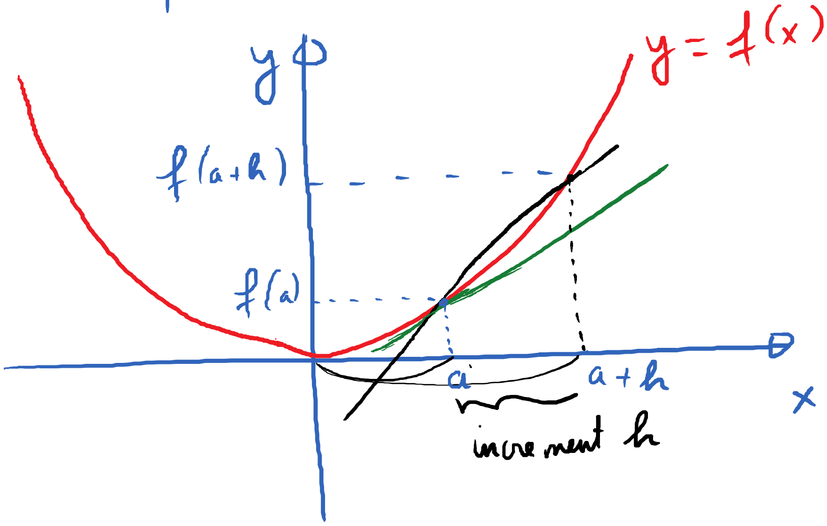
An important variation of the formula for $m_{\text{tangent}}$.




Slope of secant line

$$= \frac{f(a+h) - f(a)}{h}$$

$$m_{\text{tangent}} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$ is given by

$$m_{tangent} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

E.g. Find Slope of tangent line to the graph of $y = \sqrt{x}$ at $(1, 1)$ using the second formula.

$$m_{tangent} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}$$

$$= \lim_{h \to 0} \frac{(1+h) - 1}{h \cdot \left(\sqrt{1+h} + 1\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h} \cdot \left(\sqrt{1+h} + 1\right)} = \lim_{h \to 0} \frac{1}{\sqrt{1+h} + 1} = \boxed{\frac{1}{2}}$$

Def: The derivative of $y = f(x)$ at the point $x = a$; denoted by $f'(a)$ (read as f prime of a) or

$$\left.\frac{dy}{dx}\right|_{x=a}$$ (leibnitz notation) is defined to be

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

provided that the limit exists

**Note:** $f'(a)$ = slope of the tangent line to $y = f(x)$ at $x = a$.

**Note:** If $f$ is the position function of a moving object,
$f'(a)$ = instantaneous velocity at time $x = a$.

**E.g.** $f(x) = 8 - x - x^2$.
Find $f'(0)$ using the definition.
Solved in class.